

MATHEMATICS

Patterns and Relations

Module 2





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Mathematics 8

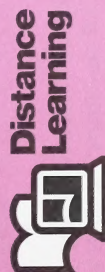
Module 2

Patterns and Relations

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Mathematics 8
Student Module Booklet
Module 2
Patterns and Relations
Learning Technologies Branch
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This document is intended for	
Students	✓
Teachers	✓
Administrators	
Parents	
General Public	
Other	

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<http://ednet.edc.gov.ab.ca/lfb>

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Welcome



Welcome to Module 2. We hope you'll enjoy your study of Patterns and Relations.

Mathematics 8 contains six modules. Work through the modules in the order given, since several concepts build on each other as you progress in the course.

Mathematics 8



The document you are presently reading is called a Student Module Booklet. You may find visual cues or icons throughout it. Read the following explanations to discover what each icon prompts you to do.



- Prepare for a problem that will provide a change of topic.



- Prepare for a challenging problem related to the topic of the activity.



- Use the Internet to explore a topic.



- Use computer software.



- Use a scientific calculator.



- View a videocassette.



- Pay close attention to important words or ideas.



- Use the suggested answers in the Appendix to correct activities.



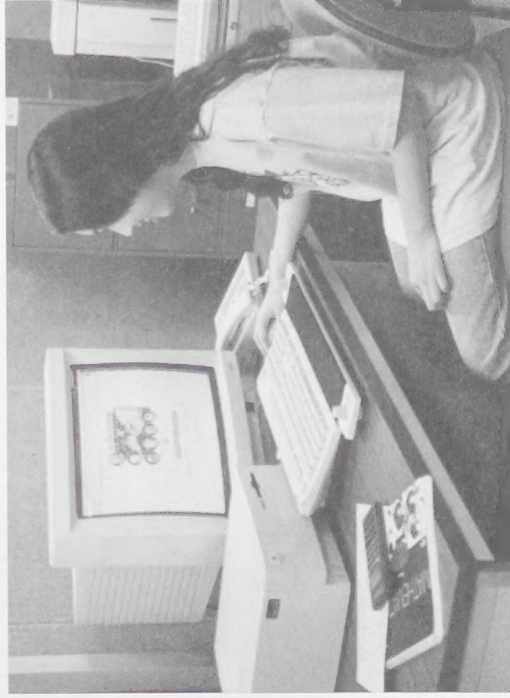
- Answer the questions in the Assignment Booklet.



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There are no response spaces provided in this Student Module Booklet. This means that you will need to use your own paper for your responses. You should keep your response pages in a binder so that you can refer to them when you are reviewing or studying.

Technology



Today society is turning to **technology** more than ever before, and it is to your advantage to be able to effectively use technology when required.



Technology is the application of tools, materials, and processes to the solution of problems. More specifically, technology refers to devices and systems that are used in processing, transferring, storing, and communicating information through electronic media.

In Mathematics 8, along with the course materials, you will use a calculator, computer, and videocassette player as tools for learning and doing mathematics.

Calculators are helpful tools for solving problems and exploring patterns and relationships between numbers. Using a calculator will also save you time and help you develop your estimating skills. Therefore, you will be given numerous opportunities in each module to use a calculator.

Computers are useful for organizing and displaying data, or drawing figures. For this reason you will have the chance in many activities to work with popular computer applications such as spreadsheets and draw programs. You will also want to check out the many Internet connections in each module.

Videocassette players allow you to view video programs on key concepts that are difficult to explain in print. That is why video programs are cited in this course.

It is expected that all of you will be able to view the video programs and use a calculator, and that most of you will do the computer activities. However, if you are unable to access a computer, you may do the calculations using a calculator, and draw figures and graphs by hand.



Problem-Solving Skills

One of the exciting features of this course is that you will develop and improve your ability in problem solving. You will need these problem-solving skills many times in your lifetime. Since this course focuses on problem solving, it is important that you understand what a **problem** is.



A problem is a task for which the method of finding the answer (as well as the answer) is not immediately known.

Like any skill, the skill of problem solving must be developed. Problems may or may not involve computation (adding, subtracting, multiplying, and dividing). Some problems are realistic; others are puzzles.

You will have the opportunity in most activities to try a problem-solving challenge. Watch for these icons.



This icon is a cue that the problem will be related to the topic of the activity.



This icon is a cue that the problem will provide a change of topic.

The Four-Stage Process

There are four stages that can be used to solve any problem: understanding the problem, developing a plan, trying the plan, and looking back.

Understanding the Problem

In this stage you should expect to feel puzzled. There are various reasons for feeling this way.

- You may not know the meanings of all the words.
- You may not understand the situation in the problem.
- You may be confused by unnecessary information.

Once you understand the problem, you should think about the problem and make an estimate of what the answer should be. This will help you arrive at a reasonable answer.

Developing a Plan

This is where you should decide on the plan of action that you are going to take to solve the problem. You may consider the following strategies:

- using objects
- using diagrams
- making a table
- working backwards
- using elimination
- using truth tables
- using an equation
- changing your point of view
- making an organized list
- using Venn diagrams
- simplifying a problem
- guessing, checking, and revising
- finding and applying a pattern
- acting out a problem

Note: The Appendix in Module 6 explains these strategies in detail. When you see a problem-solving icon in any module, you should turn to the Module 6 Appendix and review the problem-solving strategies.

Trying the Plan

In this stage you should try the plan and see if it works.

Be sure to work carefully and record your progress. You are encouraged to use a calculator to help with your calculations.



Note: While trying the plan, you should monitor your progress in order to determine if your plan will lead to a solution. You may find that the plan will not produce a solution, in which case a new plan will have to be developed.

Looking Back

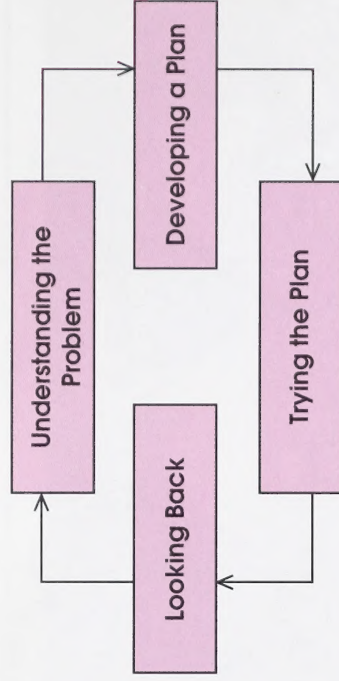
In this stage you should look back at the problem and compare your answer to the estimate you made in the first stage. Restate the problem using your answer.

Ask yourself these questions: "Did my plan work? Is my answer reasonable?"

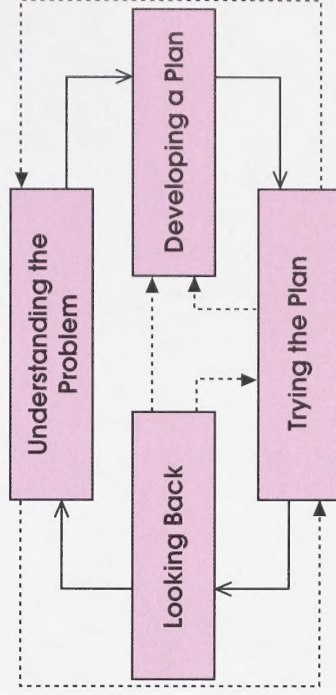
If you did not arrive at an answer, another strategy may work better. If your answer is unreasonable, you may have made errors while trying your plan.

Sequence of Stages

You usually approach a problem in the order outlined in the following diagram.



If you encounter difficulties in your original plan, or if you realize that another strategy will have better results, you may need to return to an earlier stage or use the stages in a different sequence.



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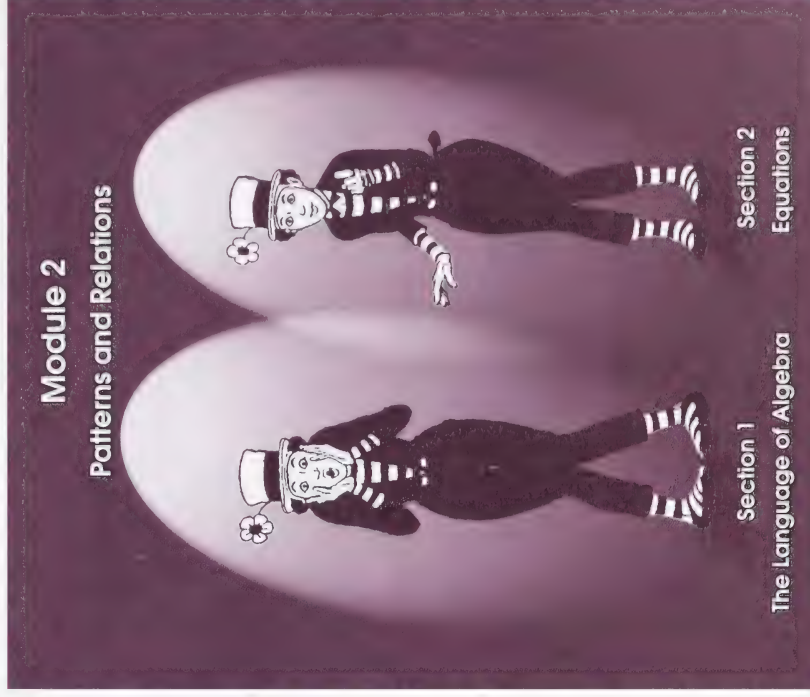
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Module Overview

Each year mimes entertain at summer festivals in many Canadian cities. As you know, mimes do not speak; they tell their stories solely through expression, gesture, and movement. The “language” of mimes is universal. This means that it is understood by children and adults from many ethnic backgrounds.

Another universal language is mathematics. Mathematics is understood by mathematicians all over the world.

In this module you will investigate a branch of mathematics called algebra. You will become familiar with the language of algebra, and you will translate English phrases into math and vice versa. You will solve equations by adding, subtracting, multiplying, and dividing. You will continue to use your problem-solving skills.



Evaluation

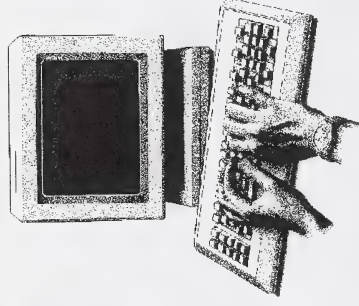
Your mark for this module will be determined by how well you complete the assignments at the end of each section and at the end of the module. In this module you must complete three assignments. The mark distribution is as follows:

Section 1 Assignment	40 marks
Section 2 Assignment	50 marks
Final Module Assignment	10 marks
TOTAL	100 marks

When doing the assignments, work slowly and carefully. You must do each assignment independently, but if you are having difficulties, you may review the appropriate section in this module booklet.



If you are working on a computer managed learning (CML) terminal, you will have a module test as well as a module assignment.



Note

There is a final supervised test at the end of this course. Your mark for the course will be determined by how well you do on the module assignments and the supervised final test.

Section 1: The Language of Algebra



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Canada has two official languages—English and French. The translators who work in the House of Commons translating the speeches of the members of Parliament must be proficient in each of the official languages.

When you solve a word problem in mathematics, you are also a translator. You take ideas expressed in words and change them into mathematical symbols. You work with these symbols. Then you express the answer to the problem in words again.

In this section you will use the language of algebra. You will translate English phrases into math and vice versa. You will work with variables, algebraic expressions, and formulas. You will graph mathematical relations. You will use algebra to generalize a pattern from a sequence of numbers or pictures. You will use formulas in spreadsheets.

Activity 1: Variables and Expressions

Canada Day, a national holiday, is a celebration of the day the Dominion of Canada was established.

The people in this photo are participating in a Canada Day celebration.

Canada celebrated its 100th birthday on July 1, 1967. What day of the week was this?



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To find what day July 1, 1967, fell on, follow these directions.

Step 1: Write the year without the century number and label it a .

$$a = 67$$

Step 2: Divide a by 4. Then write the quotient and label it b . (Ignore any remainder.)

$$b = 16$$

$$67 \div 4 = 16 \text{ R}3$$

Step 3: Use the chart at the right to write the number for the month and label it c .

Hint: Except for 1900, every year that is a multiple of 4 is a leap year.

$$c = 6$$

Step 4: Write the number of the day of the month and label it d .

$$d = 1$$

Step 5: Add a , b , c , and d . Divide this sum by 7. Write the remainder and label it e .

$$e = 6$$

$$67 + 16 + 6 + 1 = 90; 90 \div 7 = 12 \text{ R}6$$

Step 6: Refer to the following chart to find the day of the week.

Month	Number
January	6
Leap years	0
Other years	
February	2
Leap years	3
Other years	3
March	6
April	1
May	4
June	6
July	2
August	5
September	0
October	3
November	5
December	

If e is 0, the day was Sunday.
 If e is 1, the day was Monday.
 If e is 2, the day was Tuesday.
 If e is 3, the day was Wednesday.
 If e is 4, the day was Thursday.
 If e is 5, the day was Friday.
 If e is 6, the day was Saturday.

July 1, 1967, was a Saturday.

1. Use the given directions to calculate the day of the week on which each of the following events occurred.

- a. Neil Armstrong became the first person to walk on the Moon on July 20, 1969.



- b. The world's largest accidental explosion was the Halifax Explosion on December 6, 1917.
- c. The Winter Olympics opened in Calgary, Alberta, on February 13, 1988.



Check your answers by turning to the Appendix.

You may wish to use the given directions to calculate the day of the week on which you were born or on which other events you are interested in occurred.

Note: The dates must be in the 20th century.



You may use the Internet to check your answers. Following is the uniform resource locator (URL) of a site that you may find helpful.

<http://www.intellinet.com/CoolTools/CalendarMaker/>

You may also find perpetual calendars in reference books such as *The World Almanac*® and *Book of Facts*.



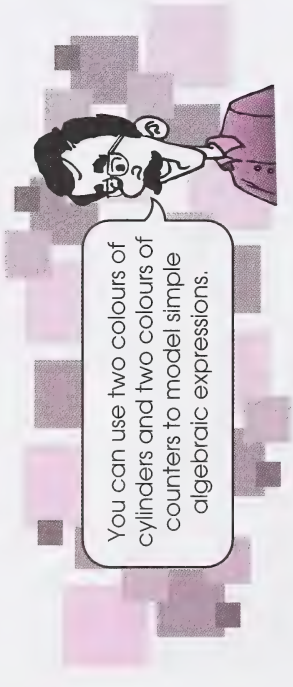
In the directions for calculating the day of the week, the letters *a*, *b*, *c*, *d*, and *e* represented the different numbers with which you had to work. These letters are called **variables**.



A variable is a quantity, represented by a symbol, that may have many values.

The branch of mathematics that uses variables to study the relationship between numbers is called **algebra**.

Algebraic expressions contain a combination of variables, numbers, and arithmetic operations. An algebraic expression may also be called a **variable expression**.



+ represents +1.

- represents -1.



represents x .



represents $-x$.

Combining two different coloured counters (or two different coloured cylinders) creates a zero pair.

The following chart lists some simple algebraic expressions and shows ways of reading the expression and a way of modelling the expression.

Expression	Ways of Reading the Expression	A Way of Modelling the Expression
$2z$	two times z twice z the product of two and z z multiplied by 2	
$x + 6$	x plus six x increased by six the sum of x and six six added to x six more than x	
$y - 5$	y minus five y take away five y decreased by five five subtracted from y five less than y	
$3 - w$	three minus w three take away w three decreased by w w subtracted from three	



In algebraic expressions, the variables act as placeholders for unknown or unspecified numbers.

An algebraic expression can be evaluated if the value of the variable is given.

Example 1

Evaluate the expression $2y$, if $y = 3$.

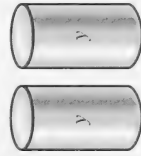
Solution

If $y = 3$, then

$$\begin{aligned} 2y &= 2(3) \\ &= 6 \end{aligned}$$

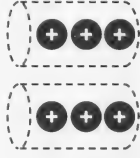
Brackets indicate multiplication.

Note: You can use learning aids to illustrate the example.



represents $2y$.

If $y = 3$, then replace each cylinder with three positive counters.



The result is six positive counters. This represents $+6$.

Example 2

Evaluate the expression $x + 6$, if $x = -1$.

Solution

If $x = -1$, then

$$\begin{aligned} x + 6 &= -1 + 6 \\ &= 5 \end{aligned}$$

Note: You can use learning aids to illustrate the example.



represents $x + 6$.

Step 1: If $x = -1$, then replace the cylinder with one negative counter.



Step 2: Simplify by removing any zero pairs.



The result is five positive counters. This represents $+5$.

Example 3

Evaluate the expression $5 - x$, if $x = -3$.

Solution

If $x = -3$, then

$$\begin{aligned} 5 - x &= 5 - (-3) \\ &= 5 + (+3) \\ &= 8 \end{aligned}$$

Note: You can use learning aids to illustrate the example.



If $x = -3$, replace the cylinder with three positive counters. **Note:** The value of $-x$ is the opposite of x .



The result is eight positive counters. This represents $+8$.



There are pages of cylinders and counters in the Appendix. Photocopy the pages, laminate the photocopies or glue them to heavy paper, and then cut out the learning aids.

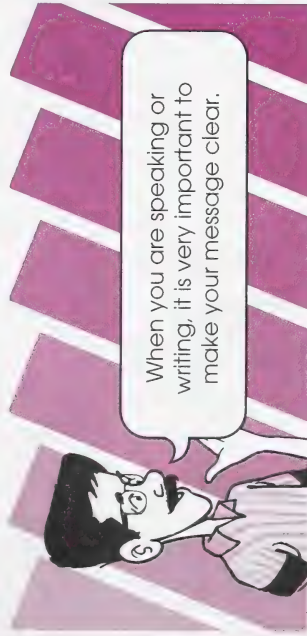
2. Translate each of the following algebraic expressions into English phrases.

- | | |
|------------|------------|
| a. $4n$ | b. $n - 3$ |
| c. $n + 2$ | d. $6 - n$ |

3. Evaluate each of the expressions in question 2, if $n = -3$. Use learning aids to illustrate your answers. **Note:** Save the learning aids. You will need them several times in this module.



Check your answers by turning to the Appendix.



Consider the following sentence: Sam said Linda I'll see you at the theatre.

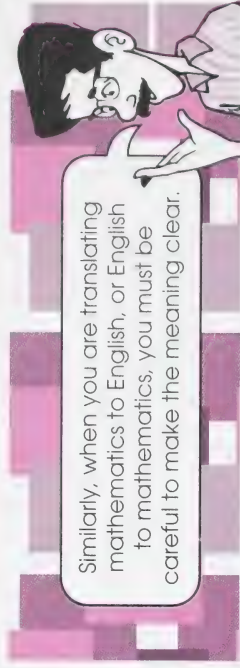
The sentence is unclear, isn't it? It can have two different meanings:

- "Sam," said Linda, "I'll see you at the theatre."
- Sam said, "Linda, I'll see you at the theatre."



Punctuation makes this written message clearer. Pauses have a similar effect in spoken messages.

You could also use different wording to make the meaning clearer.

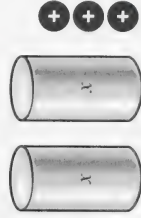


Examples 4 and 5 show how punctuation or wording can indicate the order of operations.

Example 4



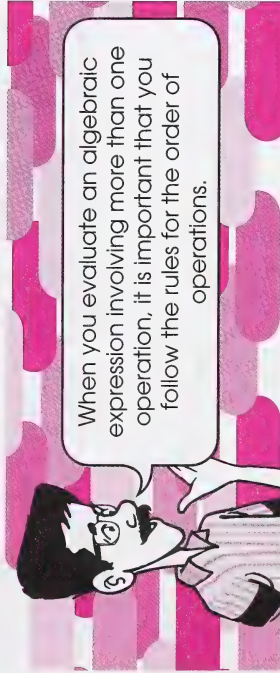
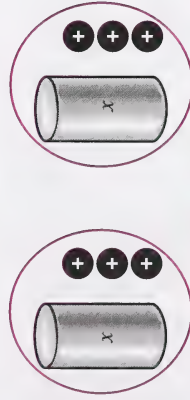
Note: The expression $2x + 3$ is modelled like this:



Example 5



Note: The expression $2(x + 3)$ means two groups of $x + 3$. It can be modelled like this:



Example 6

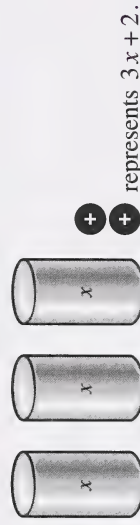
Evaluate the expression $3x + 2$, if $x = -2$.

Solution

If $x = -2$, then

$$\begin{aligned} & 3x + 2 \\ &= 3(-2) + 2 \quad \longleftarrow \text{Multiply.} \\ &= -6 + 2 \quad \longleftarrow \text{Add.} \\ &= -4 \end{aligned}$$

Note: You can use learning aids to help you visualize the example.



Step 1: If $x = -2$, replace each cylinder with two negative counters.



Step 2: Simplify by removing any zero pairs.



The result is four negative counters. This represents -4 .

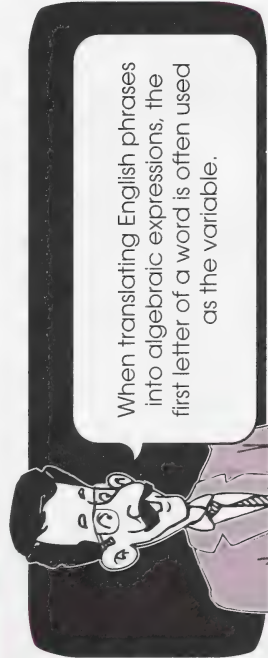
4. Translate each of the following algebraic expressions into words.

a. $2x - 5$ b. $2(x - 5)$

5. Evaluate each of the expressions in question 4, if $x = 3$. Use learning aids to illustrate your answers.



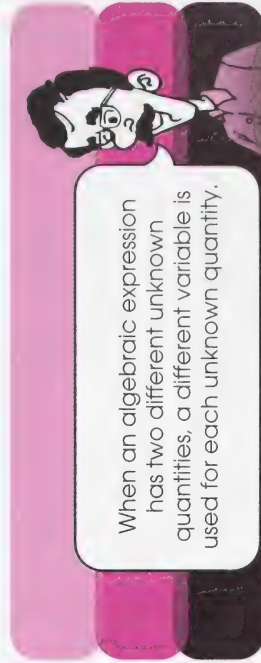
Check your answers by turning to the Appendix.



When translating English phrases into algebraic expressions, the first letter of a word is often used as the variable.

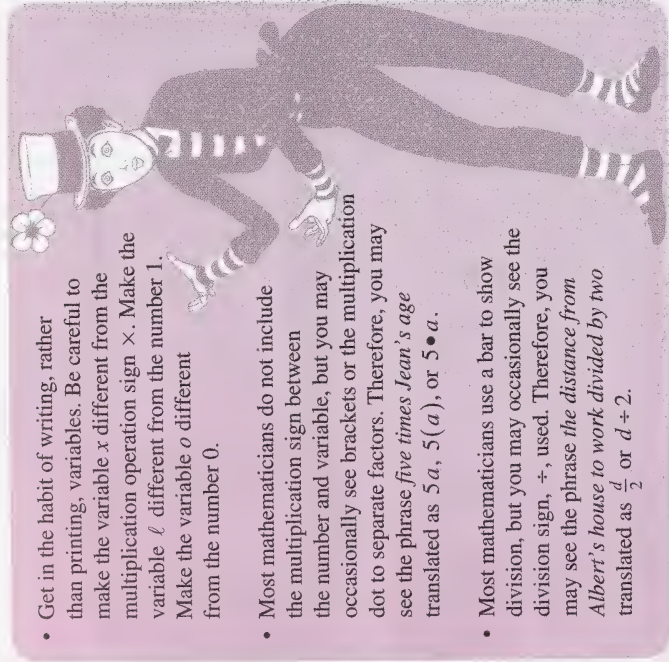
Here are some examples in which the first letter of a word is used as the variable.

Phrase	Algebraic Expression
five times Jean's a ge	$5a$
the d istance from Albert's house to work divided by two	$\frac{d}{2}$
Helen's m ass decreased by two kilograms	$m - 2$
ten dollars more than Rachel's w ages	$w + 10$
the sum of the l ength and w idth of a house	$l + w$
the number of p atients times the c ost per patient	pc
the p rice minus the d iscount	$p - d$



When an algebraic expression has two different unknown quantities, a different variable is used for each unknown quantity.

Remember the following points when translating English phrases into algebraic expressions:

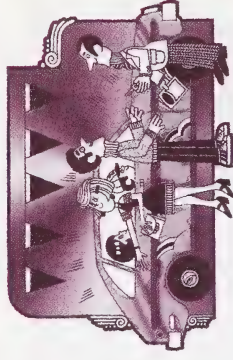


- Get in the habit of writing, rather than printing, variables. Be careful to make the variable x different from the multiplication operation sign $\times$. Make the variable ℓ different from the number 1. Make the variable o different from the number 0.
- Most mathematicians do not include the multiplication sign between the number and variable, but you may occasionally see brackets or the multiplication dot to separate factors. Therefore, you may see the phrase *five times Jean's age* translated as $5a$, $5(a)$, or $5 \bullet a$.
- Most mathematicians use a bar to show division, but you may occasionally see the division sign, $\div$, used. Therefore, you may see the phrase *the distance from Albert's house to work divided by two* translated as $\frac{d}{2}$ or $d \div 2$.

6. Translate the following phrases into algebraic expressions.

- Marco's age increased by two years
- twice the distance from Muriel's house to school

c. five times the price of a car



- a boy's age twelve years from now
- Katya's earnings divided by two
- two-thirds of the students
- three times the sum of Angela's and Jerritt's masses
- the number of words typed in a minute minus the number of errors



i. one-half of Rebecca's height, plus twice Mitchell's height



Check your answers by turning to the Appendix.

Now Try This



Use a problem-solving strategy to answer the following question.

7. Four friends went to the park to fly kites. Alexa's kite flew the highest. Baxter's kite flew 20 m lower than Alexa's but 30 m higher than Clarence's. Clarence's kite was 10 m below Dudley's. If Dudley's kite was 100 m above the ground, how high was Alexa's kite?



Check your answer by turning to the Appendix.

Did You Know?

The calendar used today is the Gregorian calendar, named after a monk who became Pope Gregory XIII. This calendar was adopted by the western world in 1582. It adjusted the leap-year pattern previously in use.

8. Use a calendar for the month of your birth to discover some interesting relationships.

- a. Draw a square around any four-by-four grid of 16 days, but do not include any blank squares.
 - i. What is the sum of the four centre numbers?
 - ii. What is the sum of the four numbers in each diagonal?
 - iii. What is the sum of the four corner numbers?
 - iv. What conclusion can you draw?
- b. Choose another four-by-four grid. Do you obtain the same results as in question 8.a.
- c. Choose a different month and repeat the steps. Do you obtain the same results?



Check your answers by turning to the Appendix.



In this activity, you investigated variables, expressions, and formulas. You continued to solve problems.

Activity 2: Formulas



Did you know that there is a simple way of calculating how far away a thunderstorm is?

Light travels about one-third of a kilometre per second. Therefore, the distance that you are from a storm can be found by counting the number of seconds between seeing a flash of lightning and hearing the resulting thunder, and then dividing this number by three.

This rule can be expressed by the following **formula**, where d is the distance (in kilometres) from the storm and t is the time (in seconds) between seeing the lightning and hearing the thunder.

$$d = \frac{t}{3}$$



A formula is an equation that shows how one quantity is related to one or more other quantities.

Following are three examples of how far away a thunderstorm is.

- How far away is the storm if you count 3 s between seeing the lightning and hearing the thunder?

If $t = 3$, then

$$\begin{aligned} d &= \frac{t}{3} \\ &= \frac{3}{3} \\ &= 1 \end{aligned}$$

The storm is 1 km away.

- How far away is the storm if you count 6 s between seeing the lightning and hearing the thunder?

If $t = 6$, then

$$\begin{aligned} d &= \frac{t}{3} \\ &= \frac{6}{3} \\ &= 2 \end{aligned}$$

The storm is 2 km away.

- How far away is the storm if you count 9 s between seeing the lightning and hearing the thunder?

If $t = 9$, then

$$\begin{aligned} d &= \frac{t}{3} \\ &= \frac{9}{3} \\ &= 3 \end{aligned}$$

The storm is 3 km away.

The formula $d = \frac{t}{3}$ expresses the relationship between time and distance. This relationship can also be shown in the following ways:

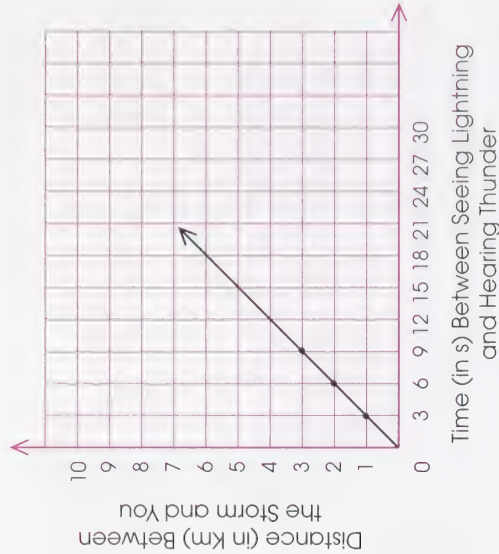
- by a table

Time (in s) Between Seeing Lightning and Hearing the Thunder	Distance (in km) Between the Storm and You
3	1
6	2
9	3
...	...

- by ordered pairs

$(3, 1), (6, 2), (9, 3), \dots$

- by a graph

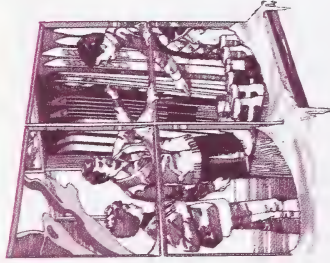


- Stephan's Ski Shoppe rents skis. The cost of renting skis is \$12.00, plus \$0.50 per hour.
This rule can be expressed by the following formula where c is the cost (in dollars) and t is the time (in hours).

$$c = 12.00 + 0.50t$$

- What is the cost of renting skis for 1 h?
- What is the cost of renting skis for 2 h?
- What is the cost of renting skis for 3 h?
- What is the cost of renting skis for 4 h?

- e. Make a graph to show the relationship between the time and cost of renting skis.



- f. How does the rental time affect the cost of renting the skis?

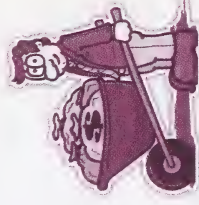
2. To quickly determine the time it will take for an investment to double, divide 72 by the interest rate.

This rule can be expressed by the following formula where t is the time (in years) and r is the interest rate (per year).

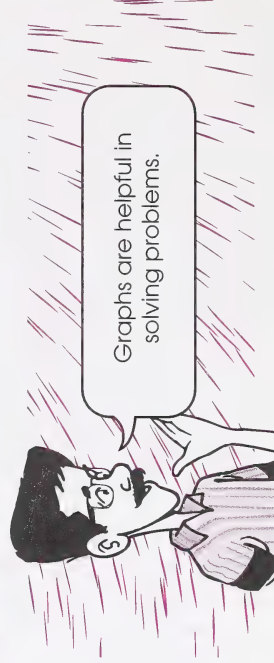
$$t = \frac{72}{r}$$

- If the interest rate is 4%, how many years will it take an investment to double?
- If the interest rate is 8%, how many years will it take an investment to double?
- If the interest rate is 12%, how many years will it take an investment to double?
- If the interest rate is 16%, how many years will it take an investment to double?

- If the interest rate is 20%, how many years will it take an investment to double?
- If the interest rate is 24%, how many years will it take an investment to double?
- Make a graph to show the relationship between the number of years it will take an investment to double and the interest rate.
- How does the interest rate affect the time it takes an investment to double?



Check your answers by turning to the Appendix.



Example



Rama gives Roger two options for the cost of printing invitations.

Option 1

a cost of \$15.00,
plus \$0.10 per
invitation

Option 2

a cost of \$0.15
per invitation

When is Option 1 less expensive?
When is Option 2 less expensive?

Solution

Step 1: Represent the unknown quantities by variables.

Let c be the cost (in dollars) of printing.
Let n be the number of invitations printed.

Step 2: Write a formula for Option 1.

$$c = 15.00 + 0.10n$$

Step 3: Write a formula for Option 2.

$$c = 0.15n$$

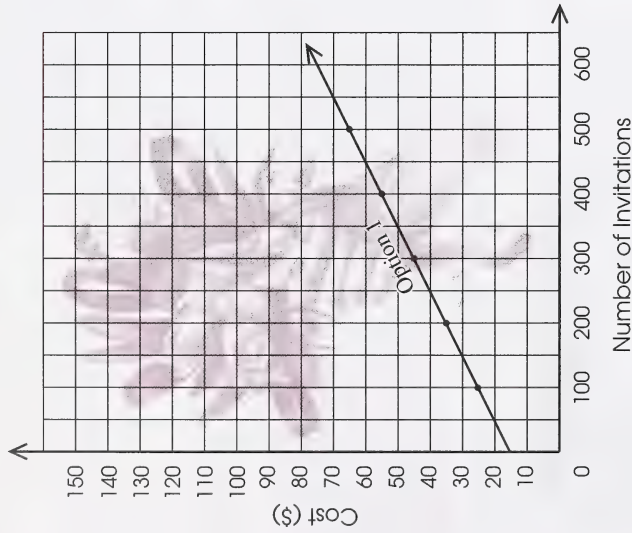
Step 4: Make a table of values for Option 1.

Option 1	
Number of Invitations	Cost (\$)
100	25
200	35
300	45
400	55
500	65

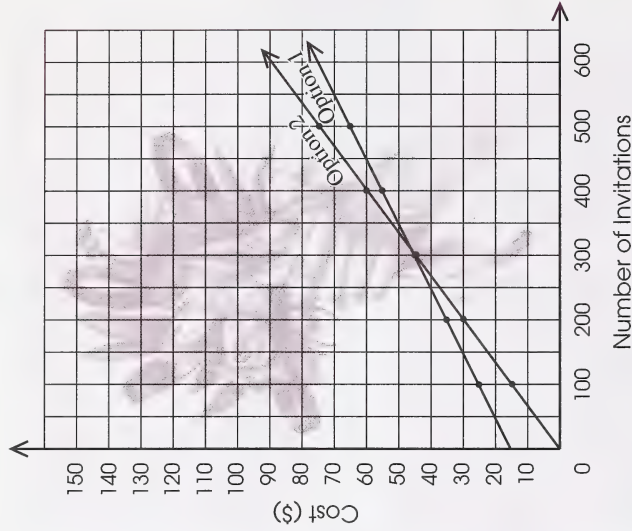
Step 5: Make a table of values for Option 2.

Option 2	
Number of Invitations	Cost (\$)
100	15
200	30
300	45
400	60
500	75

Step 6: Make a graph of Option 1. Label the graph.



Step 7: Make a graph of Option 2. Put it on the same graph as Option 1.



Step 8: Compare the graphs and answer the problem.

Option 2 is less expensive if Roger orders fewer than 300 invitations.
 Option 1 is less expensive if Roger orders more than 300 invitations.

Note: Option 1 and Option 2 are the same cost when 300 invitations are ordered.

3. Nicholas' mom wanted to encourage Nicholas to study for a test. She gave him a choice of three rewards:

<u>Reward Option 1</u> a reward of \$4 for each correct answer, minus \$5	
<u>Reward Option 2</u> a reward of \$3 for each correct answer	
<u>Reward Option 3</u> a reward of \$10, plus \$1 for each correct answer	

- There will be ten questions on the test. (Partial marks can be given.) Which option should Nicholas pick if he expected to get no correct answers? Why?
- Which option should Nicholas pick if he expected to get all ten questions correct?
- Which option should Nicholas pick if he expected to get five out of the ten questions correct?



Check your answers by turning to the Appendix.



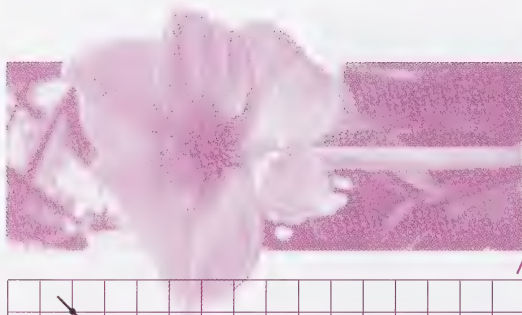
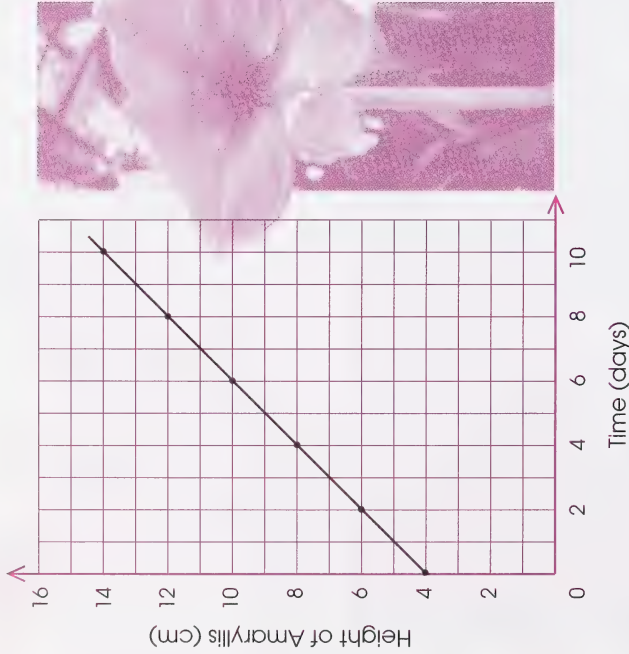
4. Ron wondered if the cooking time for a roast of beef is related to the mass of the roast. He conducted an experiment. He cooked several different-size roasts and noted the cooking time of each. He recorded this data in the following table.

Mass of Roast (kg)	Cooking Time (min)
3	120
5	200
2	80
1	40
4	160



- Ron was unable to find a pattern because the data was disorganized. Rearrange the data so that the roasts are listed in order of mass.
- Write a formula to describe how the cooking time of a roast is related to its mass. Let c be the cooking time (in min) and m be the mass (in kg).
- Use the formula to calculate the cooking time for a 4.5-kg roast.

5. Noel wondered if there is a relationship between the height of an amaryllis plant and the length of the growing time. She planted an amaryllis bulb, measured the height of the plant at regular intervals, and drew the following graph.



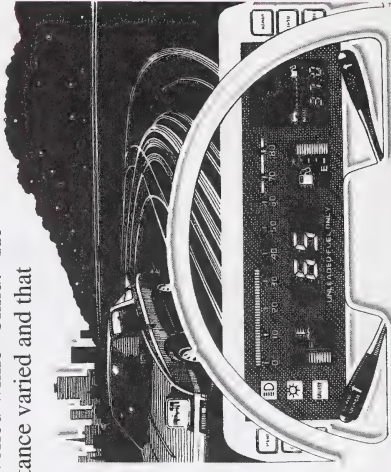
- a. Why didn't Noel put an arrowhead on her graph?

- b. Use the graph to complete the following table. **Note:** The bulb has a height as soon as it is planted because the bulb is planted with the top third above ground.

Time (days)	Height (cm)
0	
1	
2	
3	
4	
5	

- c. Write a formula to describe how the height of an amaryllis is related to its growing time. Let h be the height (in cm) and t be the time (in days).

6. When travelling by car at night a person can be “blinded” by the headlights of an oncoming vehicle. George wondered how far an average person travelled while “blind.” He discovered the distance varied and that the distance was related to the speed of the car.



Following is a table that gives data on driving while “blind.”

Speed of Car (km/h)	Distance Travelled While “Blind” (m)
0	0
30	25
60	50
90	75
120	100

- Make a graph of the data.
- Write a formula describing how the distance travelled while “blind” is related to the speed of the car. Let s represent the speed (in km/h) and d the distance (in m).
- Use the formula to predict how far a person will travel while “blind” if the car is travelling at a speed of 96 km/h.



Check your answers by turning to the Appendix.

Now Try This



Use a problem-solving strategy to answer each of the following questions.

- Yvonne and Deborah stood facing north at an intersection that was controlled by traffic lights. They watched the lights and recorded the time when the lights changed colour. They recorded this data in the following table.

Colour	Time
Green	09:01:10
Amber	09:01:34
Red	09:01:37
Green	09:02:10
Amber	09:02:34
Red	09:02:37
Green	09:03:10
Amber	09:03:34
Red	09:03:37
Green	09:04:10

The time
09:01:10
is read,
“1 minute,
10 seconds
after 9.”

- What time did the light next turn amber?
- What time did the light next turn red?
- What time did the light next turn green?
- What colour was the light at 09:06:34?
- Why do you think the light was red longer than it was green? **Hint:** Consider the traffic flow east and west.



8. Two elevators leave the seventh floor of a building at 10:00:00. The faster elevator takes 1 s to travel between floors. The slower elevator takes 2 s to travel between floors. The first elevator to reach a floor must wait for 3 s to take on passengers; the other elevator does not stop at that floor.

If Mark takes the faster elevator and Marshall takes the slower elevator, what time will each person reach the first floor?



Check your answers by turning to the Appendix.

Did you know that the world's fastest elevator can travel between 67 floors in 40 s?



You may wish to use the Internet to discover many other interesting facts about elevators. Use the search engines to explore *passenger elevators*.



In this activity, you wrote formulas to describe how quantities are related. You evaluated formulas. You graphed formulas and you analysed the graphs. You continued to solve problems.

Activity 3: Sequences

Did you know that in the Chinese culture, years are named after different animals? For example, the year 1996 was the year of the Rat. The year 1997 was the year of the Ox.



1. The years 1950, 1962, 1974, and 1986 were each the year of the Tiger. Give the next three years of the Tiger.



Check your answer by turning to the Appendix.



The numbers 1950, 1962, 1974, and 1986 form a **sequence**. Each number is 12 more than the previous number.



A sequence is a set of numbers or pictures that is ordered according to some rule or formula.

In working with sequences, it is often useful to refer to **terms**.

The first number or picture in a sequence is called the first term, the second number or picture is called the second term, and so on.



You can use patterns and the guess, check, and revise strategy to find a formula that shows how a term is related to its position in the sequence. You can use the formula to make predictions about the sequence. The Extra Help provides more information on these two strategies.

Example 1

What is the 50th term in this sequence?

2, 5, 8, 11, 14, ...

Solution

Step 1: Find a pattern in the sequence.

2, 5, 8, 11, 14, ...
+3 +3 +3 +3 +3

To obtain a term,
add 3 to the
previous term.

Step 2: Use patterns and the guess, check, and revise strategy to find a formula that tells how a term (t) is related to the position (n) of the term in the sequence.

Position of Term (n)	Term (t)
1st	2
2nd	5
3rd	8
4th	11
5th	14
...	...
n th	$3n - 1$

$t = 3 \times 1 - 1$
 $t = 3 \times 2 - 1$
 $t = 3 \times 3 - 1$
 $t = 3 \times 4 - 1$
 $t = 3 \times 5 - 1$
 $t = 3n - 1$

The formula is $t = 3n - 1$.

Step 3: Use the formula to predict the 50th term.

If $n = 50$, then

$$\begin{aligned} t &= 3n - 1 \\ &= 3(50) - 1 \\ &= 150 - 1 \\ &= 149 \end{aligned}$$

The 50th term will be 149.

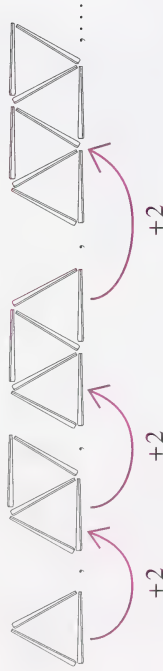
Example 2

How many toothpicks are in the 15th picture of this sequence?



Solution

Step 1: Find a pattern in the sequence.



To obtain the number of toothpicks in a picture, add 2 toothpicks to the previous picture.

Step 2: Use patterns and the guess, check, and revise strategy to find a formula relating the number of toothpicks (t) in a picture to its position (n) in the sequence.

Position of Picture (n)	Number of Toothpicks (t)
1st	3
2nd	5
3rd	7
4th	9
...	...
n th	$2n + 1$

$$\begin{aligned}
 &\longrightarrow t = 2 \times 1 + 1 \\
 &\longrightarrow t = 2 \times 2 + 1 \\
 &\longrightarrow t = 2 \times 3 + 1 \\
 &\longrightarrow t = 2 \times 4 + 1 \\
 &\longrightarrow t = 2n + 1
 \end{aligned}$$

The formula is $t = 2n + 1$.

Step 3: Use the formula to predict the number of toothpicks in the 15th picture.

If $n = 15$, then

$$\begin{aligned}
 t &= 2n + 1 \\
 &= 2(15) + 1 \\
 &= 30 + 1 \\
 &= 31
 \end{aligned}$$

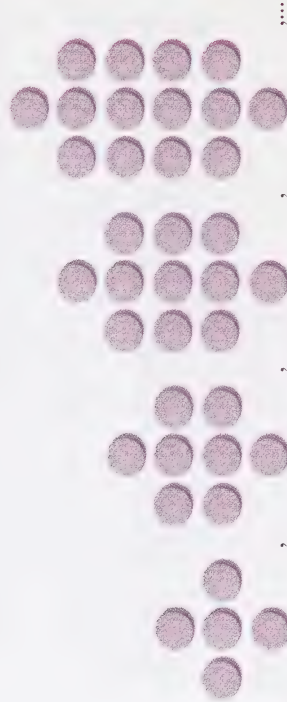
There will be 31 toothpicks in the 15th picture.

2. What is the 30th term in each of the following sequences?

a. 6, 11, 16, 21, 26, ... b. 3, 7, 11, 15, 19, ...

c. 1.5, 2, 2.5, 3, 3.5, ...

3. Predict the number of circles in the 40th picture of this sequence.



4. Predict the number of toothpicks in the 30th picture of this sequence.



Check your answers by turning to the Appendix.



You can make other formulas to describe relationships in a sequence.

Example 3

Each of the pictures in the following sequence is made up of squares and dots.



Predict the number of dots in the picture with 12 squares.

Solution

Step 1: Use patterns and the guess, check, and revise strategy to find a formula relating the number of dots (d) to the number of squares (s).

Picture	Number of Squares (s)	Number of Dots (d)
	2	3
	3	4
	4	5
	s	$s + 1$

$$d = 2 + 1$$

$$d = 3 + 1$$

$$d = 4 + 1$$

$$d = s + 1$$

The formula is $d = s + 1$.

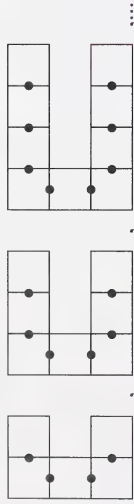
Step 2: Use the formula to predict the number of dots in the picture with 12 squares.

If $s = 12$, then

$$\begin{aligned} d &= s + 1 \\ &= 12 + 1 \\ &= 13 \end{aligned}$$

In a picture with 12 squares, there will be 13 dots.

5. Each of the pictures in the following sequence is made up of squares and dots.



Predict the number of dots in the picture with 15 squares.

6. Each of the pictures in the following sequence is made up of triangles and circles.



How many circles will there be in the picture with 15 triangles?



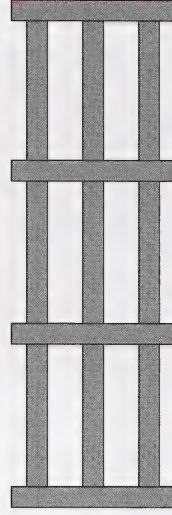
Check your answers by turning to the Appendix.



Example 4



Mary wants to make a fence that looks like this in one side of her property:



Mary decides that she will need 40 posts to build the fence. How many crosspieces will she need? (Assume there is no gate or opening in the fence.)

Solution

Step 1: Simplify the problem in order to find a pattern.

If there are 2 posts, 3 crosspieces are needed.



If there are 3 posts, 6 crosspieces are needed.



If there are 4 posts, 9 crosspieces are needed.



Step 2: Use patterns and the guess, check, and revise strategy to find a formula relating the number of crosspieces (c) to the number of posts (p).

Picture	Number of Posts (p)	Number of Crosspieces (c)
	2	3
	3	6
	4	9
	p	$3p - 3$

$$c = 3 \times 2 - 3$$

$$c = 3 \times 3 - 3$$

$$c = 3 \times 4 - 3$$

$$c = 3p - 3$$

The formula is $c = 3p - 3$.

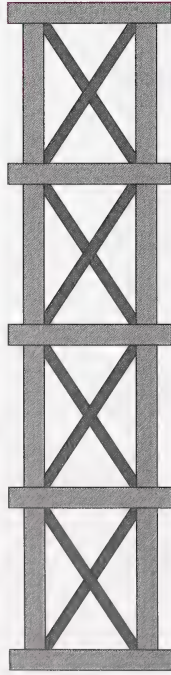
Step 3: Use the formula to find the number of crosspieces in the fence if there are 40 posts.

If $p = 40$, then

$$\begin{aligned}
 c &= 3p - 3 \\
 &= 3(40) - 3 \\
 &= 120 - 3 \\
 &= 117
 \end{aligned}$$

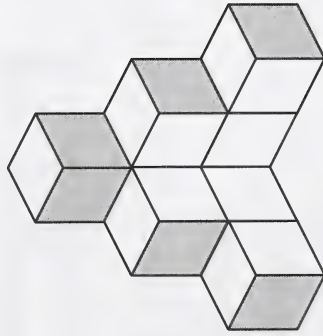
There will be 117 crosspieces.

7. Harold wants to make a fence that looks like this on one side of his property.



Harold decides that he will need 32 posts. How many crosspieces will he need? (Assume there is no gate or opening in the fence.)

8. Boxes can be stacked in a corner so that the top layer has one box and each layer has two fewer boxes than the layer below it. How many boxes would be in the bottom layer if there were 10 layers?



Check your answers by turning to the Appendix.



Now Try This

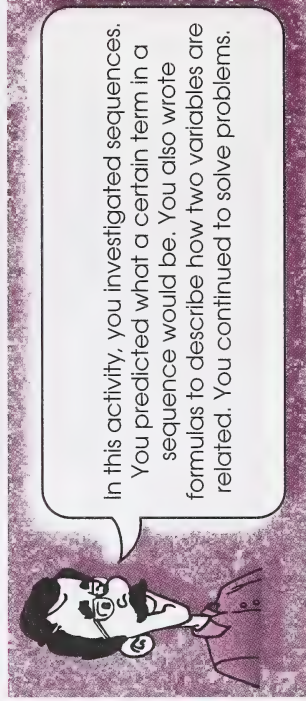


Use a problem-solving strategy to answer each of the following questions.

9. a. Write an expression equal to 10 that includes only five 9s and arithmetic signs (+, −, ×, or ÷). **Note:** Some arithmetic signs may not be used; some may be used more than once.
- b. Write an expression equal to 17 that includes only four 4s and arithmetic signs (+, −, ×, or ÷). **Note:** Some arithmetic signs may not be used; some may be used more than once.



Check your answers by turning to the Appendix.



Activity 4: Spreadsheets



Jonathan and Stephen keep statistics on the local baseball team. They use a computer and **spreadsheet** to record the information they collect.



A spreadsheet is a computer-generated arrangement of data in rows and columns.

In this activity you will use spreadsheets. (If you do not have access to a computer and a spreadsheet program, you should read the information on spreadsheets so that you are aware how computers can assist you. You should perform the operations using a calculator.)

When you open a new spreadsheet document, you see a grid of **columns** and **rows** like this:

	A	B	C	D
1				
2				
3				
4				

The top of the grid contains the column headings (labelled A, B, C,...). The left side of the grid contains the row headings (labelled 1, 2, 3, 4,...).

A **cell** is formed at the intersection of each column and row. For example, the cell at the intersection of column A and row 1 is identified as cell A1. The notation A1 is the **cell address**. You will use the cell address when you write spreadsheet formulas.

The active or selected cell is called the **current cell**. The current cell has a border around it. In a new document, cell A1 is the current cell.

You can select another cell by clicking on that cell.

You can also move around the spreadsheet in the following ways.

- The return key moves the selection one cell down.
- The tab key moves the selection one cell to the right.
- The arrow key moves the selection one cell in the direction of the arrow.

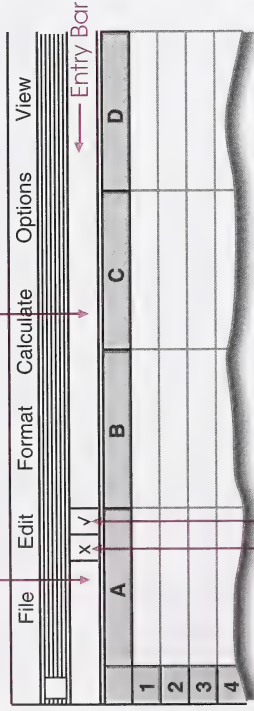


You use the **entry bar** to enter data in a spreadsheet.

The entry bar is above the column headings. It identifies the cell address and the contents of the current cell.

Cell address of current cell is shown here.

Content of current cell is shown here.



Cancel button
Accept button

To enter data in a cell, select a cell and type the data, but do **not** press "Return." The data will appear in the entry bar as you type. (If you make an error, you can backspace to make the correction.) Confirm the entry by clicking on the **accept button** or pressing "Return." When you confirm the entry, data in the entry bar is displayed in the cell.

If the data will not fit into a cell, you can resize the column. One way to do this is to drag the edge of the column heading with the mouse.



Use a computer and a spreadsheet program such as *ClarisWorks™*, *Microsoft® Works*, or *Microsoft® Excel* to answer question 1.

1. The five oldest national parks in Canada are Banff, Yoho, Glacier, Waterton, and Jasper. The following table shows the area (in hectares) of each park.

Park	Area (ha)
Banff	664 080
Yoho	131 310
Glacier	134 940
Waterton	52 590
Jasper	108 780



Start a new document in a spreadsheet program and enter this information. **Note:** Save your spreadsheet as you will use it in question 2.



Check your answer by turning to the Appendix.



Spreadsheet formulas show how two or more cells are related.

Here are some points to remember about spreadsheet formulas:

- Always begin a spreadsheet formula with an equal sign (=).
- Use cell addresses instead of variables in spreadsheet formulas.
- Use an asterisk (*) for multiplication; use a forward slash (/) for division.

For example, to find the total of the values in cells B5 and B6, you would use the following formula:

$$= B5 + B6$$

To find the product of cells A1 and B3, you would use the following formula:

$$= A1 * B3$$



Use the spreadsheet you saved in question 1 to answer question 2. Start a new document in the spreadsheet program to answer questions 3 to 5.

2. a. Find the total area of the five oldest national parks.
b. Find the average area of the five oldest national parks.

3. Katya and Zennie surveyed the colours of cars and trucks travelling west through a certain intersection for a one-hour period.

They prepared the following spreadsheet:

	A	B	C	D
1	Colour	Number of Cars	Number of Trucks	Total
2	White	15	3	18
3	Black	4	2	6
4	Red	12	5	17
5	Blue	8	6	14
6	Green	6	3	9
7	Other	9	8	17
8	Total	54	27	81
9	Average	9	4.5	6.75

- a. There is a row labelled "Total" and a column labelled "Total." What does each "Total" cell represent?
- b. How do you get the averages in row 9? (**Hint:** Give the formulas for cells B9, C9, and D9.)
- c. What formula do you use to get the total in cell D8?
- d. Name the cell in which you would find each of the following formulas. **Hint:** When a cell is selected, the formula for the cell appears in the entry bar, and the result of the calculation appears in the cell.
 - i. $= B5 + C5$
 - ii. $= C8 / 6$
 - iii. $= B2 + B3 + B4 + B5 + B6 + B7$



4. a. Explain how spreadsheet formulas are similar to algebraic formulas.
- b. Explain how spreadsheet formulas are different from algebraic formulas.
5. When some new drugs are developed and the correct dosage for adults is determined, the corresponding dosage for children may be found. For children over two years of age, the dosage may be calculated using Clark's Rule.

Clark's Rule compares a child's mass to the average adult mass of 68 kg.



This rule can be expressed by the following formula, where d is the child's dose (in milligrams), a is the adult's dose (in milligrams), and m is the child's mass (in kilograms).

$$d = \frac{am}{68}$$

Following is part of a spreadsheet. It shows the child's dose of a particular medicine for which the adult dosage is 50 mg.

	A	B	C
1	Child's Mass (kg)	Child's Dose (mg)	
2	10	7.35294117647058	
3	20	14.7058823529411	
4	30	22.0588235294117	
5	40	29.4117647058823	
6			

- a. Give the spreadsheet formula for each of the following cells.
 - i. B2
 - ii. B3
 - iii. B4
 - iv. B5
- b. Explain how you can change the format of the spreadsheet so that the numbers in Column B are rounded to the nearest whole numbers (0 decimal places).

Hint: Either experiment or read the manual for your spreadsheet program.

- c. Use your spreadsheet's charting option to graph the relationship between child's mass and the child's dose for this particular medicine.



Check your answers by turning to the Appendix.

Did You Know? 🇨🇦

Throughout this course you will be reading about the discoveries and inventions Canadians have made.

Did you know that Canadians invented the sports of hockey, basketball, and tenpin bowling? Canadian inventions also include the McIntosh apple, the first newsprint from wood fibre, the snowmobile, the electronic cooking range, and the telephone.

Find and read the article entitled “The Wheat That Created the Canadian West” in the Appendix.

6. Why were the inventions of Red Fife and Marquis wheat significant?

7. About 0.18 of Canada’s 293 000 farms are classified as wheat farms. How many farms are classified as wheat farms?



Check your answers by turning to the Appendix.



Use the Internet to discover what contribution a Canadian woman, Margaret Newton (1887–1971), made to the development of Canadian wheat. This is the uniform resource locator (URL) of a site that you may find useful.

http://www.corpserv.nrc.ca/corpserv/hall/u_m_e.html

Now Try This



Use a problem-solving strategy to answer the following question.

8. Mr. Sabiston went to a store; he spent half of his money plus \$10 more. He then went to a second store where he spent half of his remaining money plus \$10 more. If Mr. Sabiston had \$20 left, how much did he begin with?



Check your answer by turning to the Appendix.



In this activity you used spreadsheets and discovered how to write spreadsheet formulas. You continued to solve problems.

Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

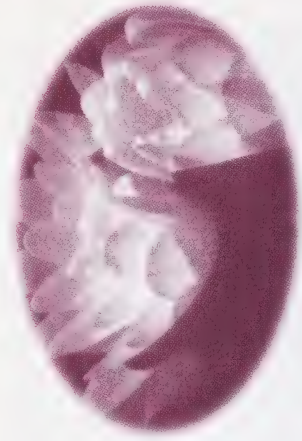
Extra Help

In this section you wrote formulas to describe how one quantity is related to one or more other quantities. You used the guess, check, and revise strategy and patterns to help you write formulas.

This Extra Help will give you more practice in writing formulas.

To write formulas using the guess, check, and revise strategy, begin with the first set of numbers. Decide one way that these numbers are related and then test to see if the other sets of numbers have the same relationship. If this relationship is not true for the other sets, revise the formula and test the others. Repeat this process until you find a formula that works for all sets of numbers.

Example

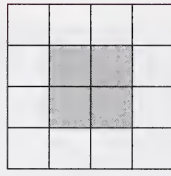


The Koeslings want to build a square lily pond in their backyard and surround it with patio blocks.

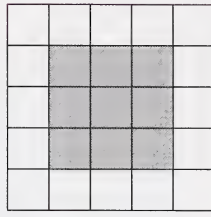
They notice a pattern:



- To build a square lily pond 1 m long, 8 patio blocks are needed.



- To build a square lily pond 2 m long, 12 patio blocks are needed.



- To build a square lily pond 3 m long, 16 patio blocks are needed.

Write a formula to describe how the number of patio blocks is related to the length of the lily pond.

Solution

Step 1: Put the given data in a table.

Length of Lily Pond (in metres)	Number of Patio Blocks
1	8
2	12
3	16

Step 2: Make a guess about how the number of patio blocks is related to the length of the lily pond and test the guess.

Is the number of patio blocks 7 more than the length of the lily pond?

Length of Lily Pond (in metres)	Number of Patio Blocks
1	8
2	12
3	16

Test

$$\begin{aligned} &\rightarrow 1 + 7 = 8 \\ &\rightarrow 2 + 7 \neq 12 \\ &\rightarrow 3 + 7 \neq 16 \end{aligned}$$

No, the number of patio blocks is not 7 more than the length of the lily pond.

Step 3: Revise the guess you made in Step 2 and test this new guess.

Is the number of patio blocks 8 times the length of the lily pond?

Length of Lily Pond (in metres)	Number of Patio Blocks
1	8
2	12
3	16

Test

$$\begin{aligned} &\rightarrow 8 \times 1 = 8 \\ &\rightarrow 8 \times 2 \neq 12 \\ &\rightarrow 8 \times 3 \neq 16 \end{aligned}$$

No, the number of patio blocks is not 8 times the length of the lily pond.

Step 4: Revise the guess you made in Step 3 and test this new guess.

Is the number of patio blocks 4 more than 4 times the length of the lily pond?

Length of Lily Pond (in metres)	Number of Patio Blocks
1	8
2	12
3	16

Test

$$\begin{aligned} &\rightarrow 4 \times 1 + 4 = 8 \\ &\rightarrow 4 \times 2 + 4 = 12 \\ &\rightarrow 4 \times 3 + 4 = 16 \end{aligned}$$

Yes, the number of patio blocks is 4 more than 4 times the length of the lily pond.

Step 5: Write a formula to describe how the number of patio blocks is related to the length of the lily pond. Let n be the number of patio blocks. Let ℓ be the length (in metres) of the lily pond.

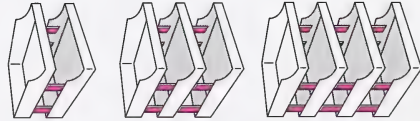
Length of Lily Pond (in metres)	Number of Patio Blocks
1	8
2	12
3	16
ℓ	$4\ell + 4$

Test

- $n = 4 \times 1 + 4$
- $n = 4 \times 2 + 4$
- $n = 4 \times 3 + 4$
- $n = 4\ell + 4$

The formula is $n = 4\ell + 4$.

1. Rosemary works in an office after school. While working with stacking file trays, she notices a pattern.



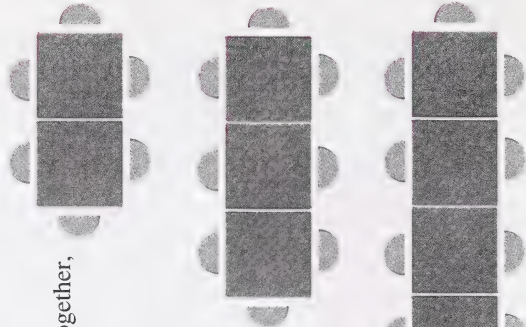
- A stack of 2 filing trays requires 4 riser rods.
- A stack of 3 filing trays requires 8 riser rods.
- A stack of 4 filing trays requires 12 riser rods.

Write a formula to describe how the number of riser rods is related to the number of trays being stacked.

2. Franco works in a restaurant. Sometimes the restaurant has banquets.



While setting up the banquet tables, Franco notices a pattern.



- If 2 square tables are put together, 6 people can be seated.
- If 3 square tables are put together, 8 people can be seated.
- If 4 square tables are put together, 10 people can be seated.

Write a formula to describe how the number of chairs is related to the number of square tables.

3. Jacintha and her friends hold a car wash to earn money for a class trip. The money the group earns is related to the number of cars washed. The earnings (after expenses are deducted) are shown in the following chart.



Number of Cars Washed	Earnings (\$)
1	0.75
2	2.00
3	3.25
4	4.50

Write a formula to describe how the earnings are related to the number of cars the group washes.



Check your answers by turning to the Appendix.

Enrichment

In this section you used graphs to show how two quantities are related. In this Enrichment you will investigate the graphs of relations that are a bit unusual. Instead of resembling a straight line or curve, these graphs will resemble steps.

Example

A parking lot attendant charges drivers a fee for parking. The charge is \$0.50 for each hour or part of an hour.

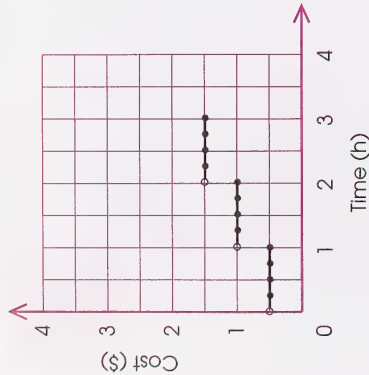
This table shows the charge for different times.

Time (h)	Cost (\$)
0.25	0.50
0.50	0.50
0.75	0.50
1.00	0.50
1.25	1.00
1.50	1.00
1.75	1.00
2.00	1.00
2.25	1.50
2.50	1.50
2.75	1.50
3.00	1.50



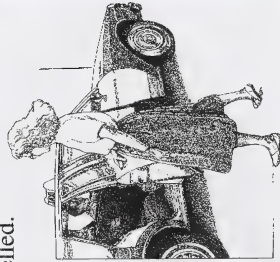
Make a graph to show the relationship.

Solution



1. A taxi driver charges a fare of \$1.50 for each kilometre or part of a kilometre.

The table at the right shows the charge for different distances travelled.



Make a graph to show the relationship between the distance travelled and the fare.

Distance Travelled (km)	Fare (\$)
0.25	1.50
0.50	1.50
0.75	1.50
1.00	1.50
1.25	3.00
1.50	3.00
1.75	3.00
2.00	3.00
2.25	4.50
2.50	4.50
2.75	4.50
3.00	4.50

2. The number of hours of sleep a school-aged child requires is related to the child's age.



This chart shows the sleeping time needed for different ages.

Age	Number of Hours of Sleep
4	12
5	12
6	12
7	11
8	11
9	11
10	10
11	10
12	10
13	9
14	9
15	9
16	8
17	8
18	8

Make a graph to show the relationship between the age of the child and the number of hours of sleep required.



Check your answers by turning to the Appendix.

Conclusion



In this section you used the language of algebra. You translated English phrases into math and vice versa. You worked with variables, algebraic expressions, and formulas. You graphed mathematical relations. You used algebra to generalize a pattern from a sequence of numbers or pictures. You used formulas in spreadsheets.

Learning the language of algebra is similar to learning a foreign language or sign language. It requires instruction and practice.

Do you know any languages other than English? Do you know sign language? Have you ever served as a translator?

Assignment



You are now ready to complete the assignment for Section 1.

Section 2: Equations

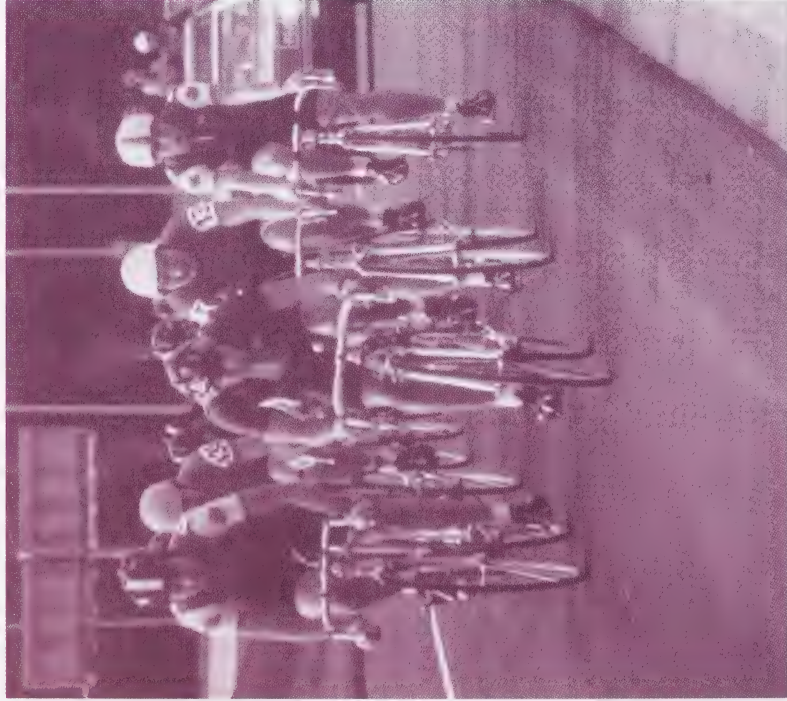


PHOTO SEARCH LTD.

Riding a bicycle is an activity that people of all ages may enjoy. Experienced riders usually can keep their balance; however, for safety reasons, bicyclists are now encouraged, and, in certain areas, required to wear helmets.

Maintaining balance is also important in solving equations. For this reason, a balance scale is used to model equations. The scale is a reminder that whenever an operation is performed on one side of an equation, the same operation must be performed on the other side of the equation.

In this section you will model, solve, and verify equations. You will use equations to solve problems. You will evaluate formulas using equation-solving skills.

Activity 1: Using One Operation to Solve Equations



Both of these sentences are true:

- Edmonton is the capital city of Alberta.
- Four more than five is nine.



Both of these sentences are false:

- Red Deer is the largest city in Alberta.
- Seven less than ten is two.



These sentences may be true or false:

- is the premier of Alberta.
- One plus gives eight.

1. Replace each to make each of the following sentences true.

- The capital of Canada is .
- The boiling point of water is °C.
- The freezing point of water is °C.
- The is the smallest bird.

2. Write each of the following sentences as equations. Let each be represented by x .

- Five plus results in thirteen.
- Four more than five is .
- Three less than eight gives the same result as plus four.
- One more than four is the same as two less than .

3. Replace each variable in the equations for question 2 with a number to make the equations true.



Check your answers by turning to the Appendix.

Each of the equations in question 3 is a **conditional equation**.



A conditional equation is an equation that is only true for certain values of the variable.

The value of the variable that makes the equation true is called the **solution**. Finding the value of the variable that makes the equation true is called **solving the equation**.

You probably solved each of the equations in question 3 by guessing, checking, and revising your guesses until you found a value that made the equation true. In this activity you will discover a more systematic method of solving equations.

To help you visualize the solving of equations, you will use a balance scale model. This model will help you remember that whenever an operation is performed on one side of an equation, the same operation is performed on the other side of the equation.



In the Appendix, find the model of a scale.

Photocopy this page, glue the photocopy to heavy paper, and cut out the scale.

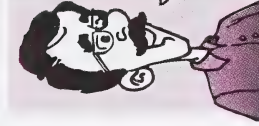


Gather this scale, the cylinders, and the counters that you used in Section 1, and the video program *Equations: Solving with One Step* from the series *Math Moves*.

4. View the video program and do the video assignments. **Note:** Keep the counters; you will need them in Module 3.



Check your answers by turning to the Appendix.



In the video you worked with equations of the form $x + a = b$ and $ax = b$, where x is a variable and a and b are integers.

You will now discover how you can solve these two types of equations using a paper-and-pencil method.

Study the following examples. In each example the modelling method of solving the equation is shown first. The paper-and-pencil method is shown second. Notice the similarity between the two methods.

Example 1

Solve the equation $w - 3 = -2$ and verify the solution.

Solution

Method 1: Using Models

To solve the equation $w - 3 = -2$, first model it.

Subtracting 3 from w is the same as adding -3 to w .



Ask yourself, "How can I isolate w ?"

You can isolate w by undoing the addition of -3 . That is, add $+3$ to **each** side of the equation.



Simplify **each** side by removing any zero pairs.



This is the result; w has been isolated.



$$\therefore w = 1$$

Method 2: Using the Paper-and-Pencil Method

To solve the equation $w - 3 = -2$, ask yourself, “How can I isolate w ?”

You can isolate w by undoing the subtraction of 3. That is, add $+3$ to **each** side of the equation.

$$\begin{array}{r} w - 3 = -2 \\ +3 \quad +3 \\ \hline w = 1 \end{array}$$

Verification

Method 1: Using Models

To verify the solution, first model the equation $w - 3 = -2$.



Replace w with one positive counter.



Simplify the left-hand side by removing the zero pairs.



The left side equals the right side.

Method 2: Using the Paper-and-Pencil Method

LS	RS
$w - 3$	-2
$= w + (-3)$	
$= 1 + (-3)$	
$= -2$	
LS	= RS

Example 2

Solve the equation $2r = 6$ and verify the solution.

Solution

Method 1: Using Models

To solve the equation $2r = 6$, first model it.



Notice that the cylinders are alone on one side of the equation; however, there are two cylinders, not one.

To solve for r , first arrange the learning aids on **each** side of the equation into two equal groups.



Examine **one** group from each side.



$$\therefore r = +3$$

Method 2: Using the Paper-and-Pencil Method

To solve the equation $2r = 6$, ask yourself, "How can I get $1r$ on the left side of the equation?"

You can get $1r$ on the left side of the equation by dividing **each** side of the equation by 2.

$$\frac{2r}{2} = \frac{6}{2}$$

$$r = 3$$

Alternatively, you can get $1r$ on the left side of the equation by multiplying each side of the equation by $\frac{1}{2}$.

$$\frac{1}{2} (2r) = \frac{1}{2} (6)$$

$$r = 3$$

Verification

Method 1: Using Models

To verify that $r = 3$, first model the equation $2r = 6$.



Replace **each** cylinder with three positive counters.



The left side equals the right side.

Method 2: Using the Paper-and-Pencil Method

$2r$	6
$= 2(3)$	
$= 6$	

$LS = RS$

RS

5. In each of the following equations what operation should be performed to solve the equation?

- a. $x + 2 = 7$

b. $t - 5 = 1$

c. $m + 3 = -1$

d. $y - 2 = -8$

e. $3b = -6$

f. $-2s = -12$

6. Solve each of the equations in question 5. Illustrate your answers by using models.

7. Verify each of the solutions in question 6. Illustrate your answers by using models.



Check your answers by turning to the Appendix.

You have examined equations in which a number has been added to the variable or subtracted from the variable. You have also investigated equations in which a variable has been multiplied by a number. You are now ready to consider equations in which the variable has been divided by a number.

Verification

LS	RS
$\frac{x}{3}$	5
$= \frac{15}{3}$	
$= 5$	

LS	=	RS
----	---	----

8. Solve each of the following equations. Verify each solution.

a. $\frac{n}{4} = 3$ b. $\frac{m}{2} = -3$



Check your answers by turning to the Appendix.

Solution

To solve the equation $\frac{x}{3} = 5$, ask yourself, "How can I get $1x$ on the left side of the equation?"

You can get $1x$ on the left side of the equation by multiplying **each** side of the equation by 3.

$$3 \left(\frac{x}{3} \right) = 3 (5)$$

$$x = 15$$



Example 4

Fatima received an allowance of \$25. She deposited this money in her savings account. If Fatima's new balance was \$81, how much was her previous balance?

Note: Check your answer.

Solution

Step 1: Identify the unknown number or quantity and choose a variable to represent it.

Let the previous balance be p .

Step 2: Write an equation to describe the situation.

$$p + 25 = 81$$

Step 3: Solve the equation and answer the question.

Ask yourself, "How can I isolate p ?" You can isolate p by adding -25 to **each** side of the equation.

$$\begin{array}{rcl} p + \cancel{25} & = & 81 \\ \cancel{-25} & -25 & \\ \hline p & = & 56 \end{array}$$

Fatima's previous balance was \$56.

Check

Determine if the answer is reasonable.

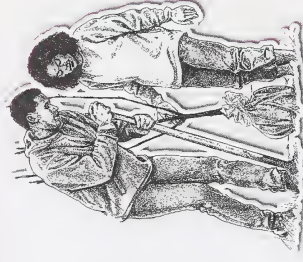
Ask yourself, "Would Fatima have a total of \$81 if she added \$25 to \$56?"

Yes, the answer is reasonable.

Note: When checking, do not merely verify the solution of the equation; you may have written the wrong equation. Instead, reread the problem and substitute your solution to see if the answer is reasonable.

Example 5

Vijay had a part-time summer job with a landscape firm. She was paid \$5 an hour. If Vijay earned \$40 the first week, how many hours did she work? **Note:** Check your answer.



Solution

Step 1: Identify the unknown number or quantity and choose a variable to represent it.

Let the number of hours Vijay worked be n .

Step 2: Write an equation to describe the situation.

$$5n = 40$$

Step 3: Solve the equation and answer the question.

Divide each side by 5 or multiply each side by $\frac{1}{5}$.

$$\frac{5n}{5} = \frac{40}{5} \quad \text{or} \quad \frac{1}{5}(5n) = \frac{1}{5}(40)$$

$$n = 8 \quad n = 8$$

Vijay worked 8 h the first week.

Check

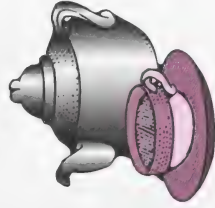
Determine if the answer is reasonable.

Ask yourself, "Would Vijay earn \$40 if she worked 8 h and was paid at \$5 per hour?"

Yes, the answer is reasonable.

Use questions to answer questions 9 to 14. Determine if each answer is reasonable.

9. Joe had 5 tea bags. He bought a new box of tea bags. If he now had 125 tea bags, how many tea bags were in the box Joe purchased?



10. Chandia used an average of 6 sheets of loose-leaf paper per day. At the end of the term she had used 654 sheets of paper. How many days long was the term?

11. Jeannette bought two train tickets for \$23. One ticket was a child's ticket and the other an adult's ticket.

If the child's ticket cost \$5, how much did the adult ticket cost?



12. The width of a rectangular card is 8 cm less than the length. If the width is 12 cm, what is the length?

13. Maria had a length of fabric to make banners. She cut the fabric into six equal pieces. If each piece was 2.75 m long, what was the original length of the fabric?



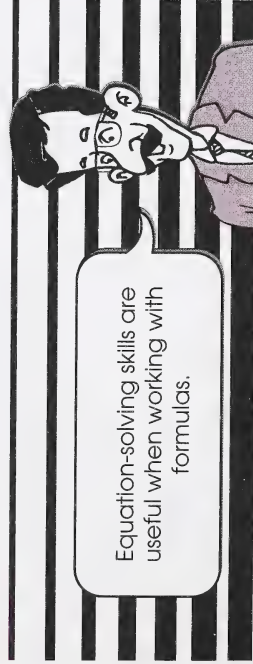
14. Hans gave Vern $\frac{1}{2}$ of his baseball cards. If Hans then had 36 baseball cards, how many cards did he have originally?



Check your answers by turning to the Appendix.



Use the Internet to discover more about sports collectables such as baseball cards.



Example 6

The following formula shows how the distance travelled is related to the speed and the time travelled.

$$d = st$$

The variable d gives the distance travelled (in km), s gives the speed (in km/h), and t gives the time (in h).

If the Mudryks and their friends drove 360 km in 4 h, at what average speed was their car travelling?



Solution

Step 1: Replace the variables with the given values and simplify the equation.

$$\begin{aligned} d &= st \\ 360 &= s(4) \\ 360 &= 4s \end{aligned}$$

Step 2: Solve the equation and answer the question.

Divide each side by 4 or multiply each side by $\frac{1}{4}$.

$$\frac{360}{4} = \frac{4s}{4} \quad \text{or} \quad \frac{1}{4} (360) = \frac{1}{4} (4s)$$

$$90 = s \qquad 90 = s$$

The car was travelling at an average speed of 90 km/h.

- 15.** Kangaroos are the only large mammals that hop. The distance they hop is related to the speed at which they are travelling.

The following formula shows how the distance between hops (d) in metres is related to their speed (s) in kilometres per hour.

$$d = 0.12s$$

- If a kangaroo covers 2.4 m in each hop, at what speed is it travelling?
- If a kangaroo covers 4.8 m in each hop, at what speed is it travelling?



- 16.** When money is borrowed the amount of money paid back is equal to the principal plus the interest. Principal is the amount of money borrowed. Interest is the charge for the use of the money.

The following formula shows how the amount of money paid back (A) is related to the principal (P) and the interest (I).

$$A = P + I$$

Find the interest if the amount paid back was \$1462 and the principal was \$1250.

- 17.** The speed of a train (in km/h) can be calculated by counting the clicks you hear in 25 s and dividing this number by 2. This rule can be expressed by the following formula.

$$s = \frac{c}{2}$$

The variable s gives the speed of the train (in km/h), and c is the number of clicks that you hear in 25 s.

If a train is travelling at 80 km/h, how many clicks should you hear in 25 s?



Check your answers by turning to the Appendix.

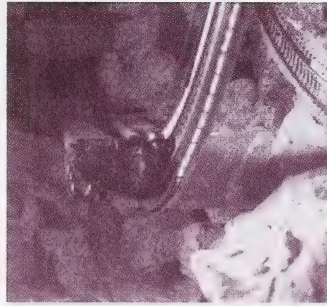
Now Try This



Use a problem-solving strategy to answer the following question.

18. A model train that is 1 m long and travelling at a speed of 1 m/s comes to a tunnel that is 1 m in length. How long will it take the train to travel completely through the tunnel?

Hint: Use the formula $d = st$, where d is the distance travelled (in m), s is the speed (in m/s), and t is the time (in min).



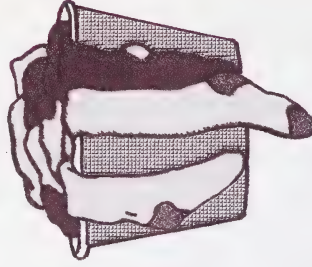
Check your answer by turning to the Appendix.



Use a problem-solving strategy to answer each of the following questions.

19. Freddy enters his dark bedroom. He takes some socks out of a laundry basket, but he cannot see the colour of the socks in the dark.

If there are two colours of socks in the basket, how many socks must Freddy take to be sure of getting a matching pair?



20. Albert Green, Ron Black, and Billy White went to a ball game. One boy wore a green shirt, one boy wore a black shirt, and one boy wore a white shirt.

Billy said, "Isn't it odd that our shirts match our last names, but not one of us is wearing a shirt that matches his own name?"

"That's true," said the boy in the black shirt.

What colour is each boy's shirt?



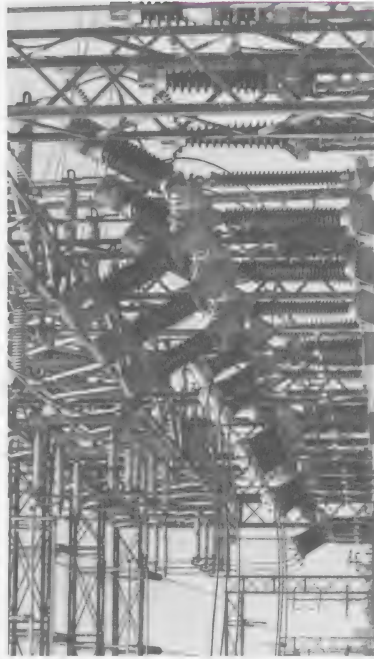
Check your answers by turning to the Appendix.

Did You Know?

Canadians have made many significant discoveries and inventions. Did you know that Canadians made advances in the transmission of electricity?

Find and read the article entitled “The First Transmission of Electrical Power at 735 kV” in the Appendix.

Note: The unit kV is read as “kilovolts.” A volt is a unit of electrical pressure; 735 kV equals 735 000 volts.



Use the Internet or other sources to find further information on electrical power. Many provincial power corporations (for example, TransAlta Corporation, B.C. Hydro, and Manitoba Hydro) have sites on the Internet. You may wish to e-mail the various companies to discover more about specific topics that interest you.

21. In 1997, TransAlta Utilities charged 7.58¢/kWh for the electrical power a household consumed. **Note:** The rate of 7.58¢/kWh is read as “7.58 cents per kilowatt hour.” To give you a sense of how much energy 1 kWh is, you can think of it as the amount of energy used by ten 100-watt light bulbs in 1 h.

- a. Write a formula to show the relationship between the cost of energy in dollars and the amount of energy consumed. Let c represent the cost and e represent the energy used.
Hint: 100¢ = \$1.
- b. Use the formula to calculate the cost if 638 kWh of electricity are used by a family.

22. List five ways that you can be an energy saver.



Check your answers by turning to the Appendix.



In this activity you modelled equations. You discovered that some equations can be solved by performing one operation to each side of the equation. You used equation-solving skills to work with formulas.

Activity 2: Using Two Operations to Solve Equations



Models are often used in the everyday world to help people visualize concepts. For example, a globe may be used to give people a sense of where places are located.

In this activity you will continue to use learning aids to help you visualize the solving of equations.

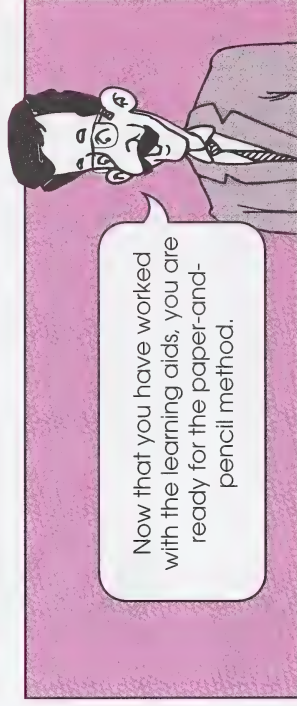


Gather your equation scale, cylinders, counters, and the video program *Equations: Solving with More than One Step* from the series *Math Moves*.

1. View the segment entitled “Equations of the Form $ax + b = c$ ” and do the video assignments. (You do not need to watch the other segments on the video.)



Check your answers by viewing the video.



Study the following example. The modelling method of solving the equation is given first. The paper-and-pencil method is shown second. Notice the similarity between the two methods.

Example 1

Solve the equation $2m - 1 = 7$ and verify the solution.

Solution

Method 1: Using Models

Step 1: Isolate the variable.

First model the equation.

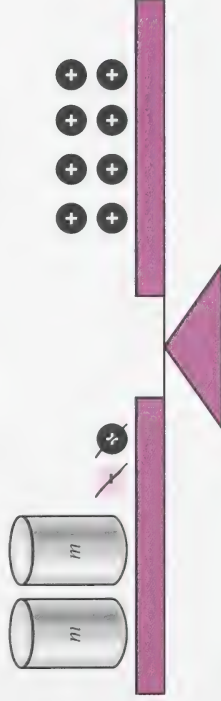


Ask yourself, "How can I isolate the cylinders?"

You can isolate the cylinders by undoing the addition of one negative counter. That is, add one positive counter to **each** side of the equation.



Simplify the equation by removing any zero pairs.

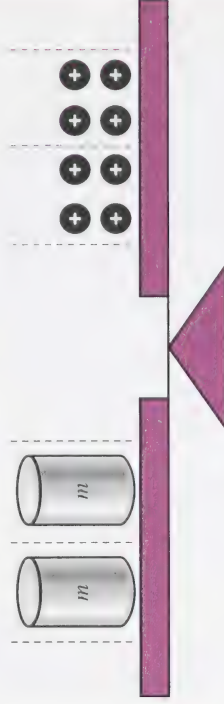


The result is this. The cylinders have been isolated.



Step 2: Solve for m .

First arrange the learning aids on **each** side of the equation into two equal groups.



Then examine **one** of the two equal groups on **each** side of the equation. This is the result.



$$\therefore m = 4$$

Method 2: Using the Paper-and-Pencil Method

Step 1: Isolate the variable.

Add +1 to each side.

$$\begin{array}{r} 2m - 1 = 7 \\ +1 \quad +1 \\ \hline 2m = 8 \end{array}$$

Step 2: Solve for m .

Divide each side by 2.

$$\begin{array}{r} \frac{2m}{2} = \frac{8}{2} \\ m = 4 \end{array}$$

or

Multiply each side by one-half.

$$\begin{array}{r} \frac{1}{2} (2m) = \frac{1}{2} (8) \\ m = 4 \end{array}$$

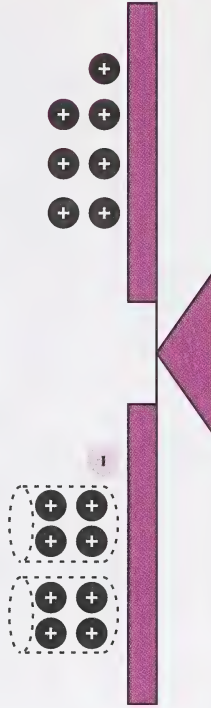
Verification

Method 1: Using Models

To verify the solution, first model the original equation.



Replace **each** cylinder with four positive counters.



Simplify the equation by removing any zero pairs.



This is the result.



The left side equals the right side.

Method 2: Using the Paper-and-Pencil Method

LS	RS
$2m - 1$	7
$= 2(4) - 1$	
$= 8 - 1$	
$= 7$	

LS = RS

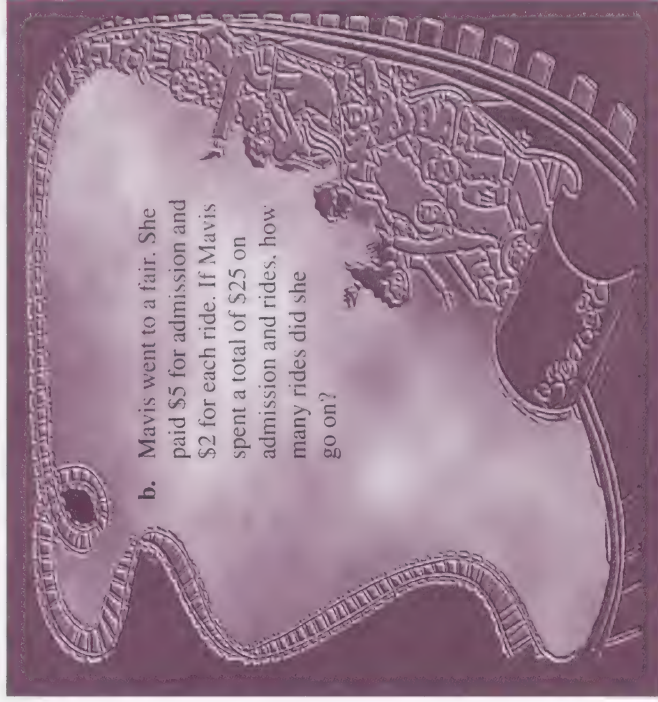
2. Solve each of the following equations. Use models to confirm your answers.

- a. $3x + 1 = 10$ b. $2y - 3 = 5$
 c. $3z - 1 = -10$ d. $4 - a = 6$

3. Verify each of the solutions in question 2. Use models to confirm your answers.

4. Use equations to answer each of the following problems.

- a. The difference between Effi's age and Avi's age is 12 years. Effi is older than Avi. If Effi is 29 years old, how old is Avi?



- b. Mavis went to a fair. She paid \$5 for admission and \$2 for each ride. If Mavis spent a total of \$25 on admission and rides, how many rides did she go on?

- c. Margaret bought 3 identical bags of cookies. She shared the cookies with the teens in her youth group. The teens ate 39 cookies. If there were 15 cookies left, how many cookies were originally in each bag?

- d. Five more than three times yesterday's high temperature was -31°C . What was yesterday's high temperature?



- e. Troy had \$24 in his bank account. He made 4 equal deposits. His bank account then had a balance of \$72. What was the amount of each deposit?

- f. It costs \$3 plus \$2 per hour to rent skates at the park. If Maria has \$15, what is the maximum time that she can rent skates?



5. It has been found that, on average, when children are between the ages of 20 months and 50 months, the size of their vocabulary is related to their age.



The following formula gives this relationship. In this formula, n is the number of words the child can use correctly and a is the age of the child in months.

$$n = 60a - 900$$

At what age (in months) does a child use 600 words correctly?



Check your answers by turning to the Appendix.



When solving equations involving decimal numbers, you may use a calculator.



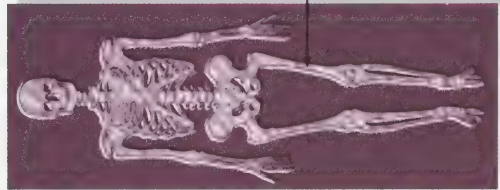
Use a calculator to answer question 6.

6. Scientists can estimate the height of an adult by measuring the length of the person's femur (thighbone).

- a. The following formula shows how the height of an adult male is related to the length of his femur (in centimetres). In the formula, h is the person's height (in centimetres) and f is the length of the femur (in centimetres).

$$h = 1.88f + 81.3$$

The tallest man in medical history was Robert Wadlow of the United States. When Wadlow was measured in 1940 he was 272 cm tall. How long would his femur have been? Express the answer to the nearest centimetre.



- b. The following formula shows how the height of an adult female is related to the length of her femur. In the formula, h is the person's height (in centimetres) and f is the length of the femur (in centimetres).

$$h = 1.95f + 73$$

The tallest woman in medical history was Zeng Jinlian of China. When she was measured in 1982 she was 248 cm tall. How long would her femur have been? Express the answer to the nearest centimetre.



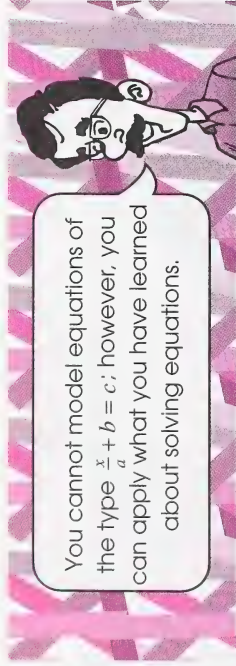
Check your answers by turning to the Appendix.



Use the Internet and other sources to find more information about Robert Wadlow and Zeng Jinlian.



You will now solve equations of the form $\frac{x}{a} + b = c$, where x is a variable, and a , b , and c are integers.



You cannot model equations of the type $\frac{x}{a} + b = c$; however, you can apply what you have learned about solving equations.

Example 2

Solve $\frac{b}{4} - 1 = 2$ and verify the solution.

Solution

Step 1: Isolate the variable.

Ask yourself, “How can I isolate the variable?” You can add +1 to **each** side of the equation.

$$\begin{array}{rcl} \frac{b}{4} - 1 & = & 2 \\ \frac{b}{4} - 1 + 1 & = & 2 + 1 \\ \frac{b}{4} & = & 3 \end{array}$$

Step 2: Solve for b .

Multiply **each** side of the equation by 4.

$$\begin{array}{rcl} 4 \left(\frac{b}{4} \right) & = & 4 (3) \\ b & = & 12 \end{array}$$

Verification

LS	RS
$\frac{b}{4} - 1$	2
$= \frac{12}{4} - 1$	
$= 3 - 1$	
$= 2$	
LS	= RS

7. Solve each of the following equations; then verify each solution.

a. $\frac{n}{6} + 1 = 2$

b. $\frac{d}{3} - 5 = 1$

c. $3 - \frac{m}{2} = 9$

d. $5 - \frac{r}{2} = -11$

8. Use equations to solve each of the following problems.



a. Vancho gave Vera half of his paper clips. Vera lost 7 of the paper clips Vancho gave her, but she had 23 left. How many paper clips did Vancho have originally?

b. Shannon’s present age is 2 years more than one-third of her father’s age. If Shannon is 14 years old, how old is her father?

- c. On average, the mass of a raccoon is 1 kg more than one-half of the mass of a coyote. The average mass of a raccoon is 9 kg. Find the average mass of a coyote.



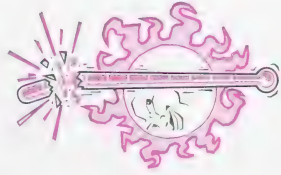
9. The number of chirps a cricket makes is related to the temperature.

The following formula shows how you can determine the temperature by counting the number of chirps a cricket makes in a minute.

$$t = \frac{n}{8} + 5$$

In the formula, t represents the temperature (in degrees Celsius) and n represents the number of chirps the cricket makes in a minute.

How many times a minute should a cricket chirp at a temperature of 25°C ?



10. A lake is 20 m deep at its centre. At the surface, the water is 15°C .

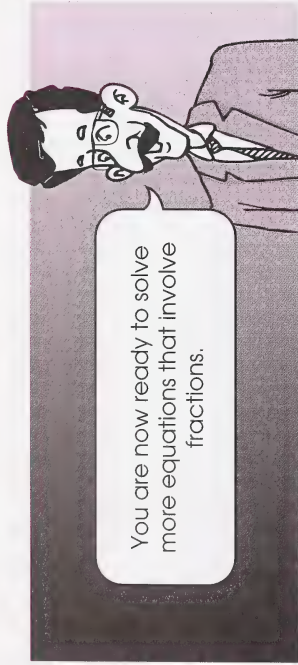
The following formula shows how the temperature of the water in this lake is related to the depth of the water. In the formula, d represents the depth (in metres) and t represents the temperature (in degrees Celsius).

$$t = 15 - \frac{d}{4}$$

At what depth is the water 10°C ?

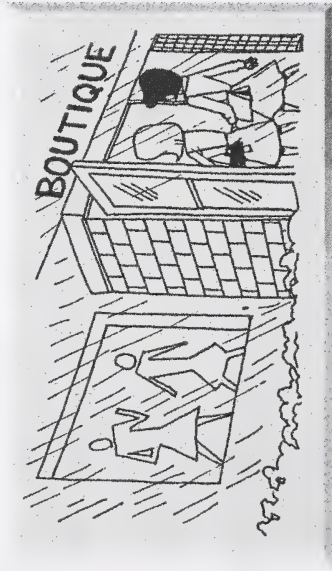


Check your answers by turning to the Appendix.



Example 3

At a local boutique dresses are on sale for $\frac{2}{3}$ of their regular price.



The sale price of a dress is \$80. What is the regular price of the dress? **Note:** Check your answer.

Solution

Step 1: Write an equation.

Let r be the regular price (in \$).

$$\frac{2r}{3} = 80$$

Step 2: Clear the denominator by multiplying **each** side of the equation by 3.

$$3\left(\frac{2r}{3}\right) = 3(80)$$

$$2r = 240$$

Step 3: Find the value of r .

$$\frac{2r}{2} = \frac{240}{2}$$

$$r = 120$$

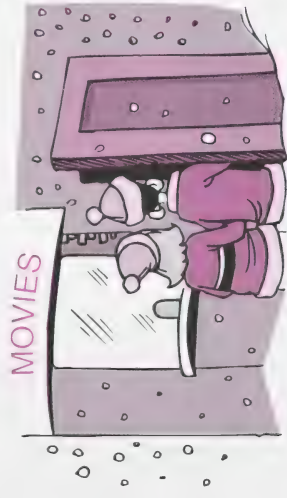
The regular price of the dress was \$120.

Check

Is $\frac{2}{3}$ of \$120 equal to \$80?

Yes, the answer is reasonable. The regular price of the dress was \$120.

11. At matinee performances the cost of admission to a movie is three-fourths of the regular price. If Lloyd and Heather each pay \$6 for admission to a matinee performance, what is the regular price?



12. Sweaters are on sale for one-third off the regular price. If the sale price is \$12, what is the regular price? **Hint:** $\frac{1}{3}$ off is the same as $\frac{2}{3}$ of the regular price.



Check your answers by turning to the Appendix.



Now Try This



Use a problem-solving strategy to answer the following question.

13. a. Use some of the following information to make up a problem.

It is 300 km from Regina to Gull Lake. About halfway between the two locations is Chaplin. Deleho drives her car the speed limit of 100 km/h. Alain drives his car 10 km/h slower than Deleho.

- b. Use an equation to solve the problem.



Check your answers by turning to the Appendix.

Did You Know?

In cryptography, letters are used to represent numbers and other letters to ensure the secrecy of a message. The coded message is called a **cryptogram**. A mathematical cryptogram, one in which letters represent only numbers, may also be called a **cryptarithm**.

In question 14 you will decode two cryptograms by using logical reasoning.

14. Isobel's parents love puzzles. While at summer camp, Isobel sent a postcard with the following note to them.

$$\begin{array}{r} \text{SEND} \\ + \text{ MORE} \\ \hline \text{MONEY} \end{array}$$

Her parents replied with this message.

$$\begin{array}{r} \text{SPEND} \\ - \text{ LESS} \\ \hline \text{DEAR} \end{array}$$


JIM WHITMER PHOTOGRAPHY

For each puzzle, find a number sentence that makes it true. A letter repeated within a puzzle represents the same digit, but it may be a different digit in the other puzzle.



Check your answers by turning to the Appendix.



In this activity you solved two-step equations. You used equation-solving strategies to answer problems and solve formulas.

Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

In this section you used the process of undoing operations to solve equations. For example, in the equation $5x = 15$, x is multiplied by 5. To undo the multiplication, you can divide each side by 5.

Doing and undoing operations also occur in the everyday world. An example of a doing action is tying your shoes. Untying your shoes brings them back to their original state.



1. In a chart like this, write a phrase that undoes each of the given actions and brings it back to the original state.

Doing Action	Undoing Action
Open a door.	
Turn on a tap.	
Put on socks and then put on shoes.	
Unload cargo from a truck and put cargo on a boat.	



Check your answers by turning to the Appendix.

Flow charts and reverse flow charts are helpful in showing the order of doing actions and undoing actions.

Example 1

What actions must be performed to undo the following sequence of actions?

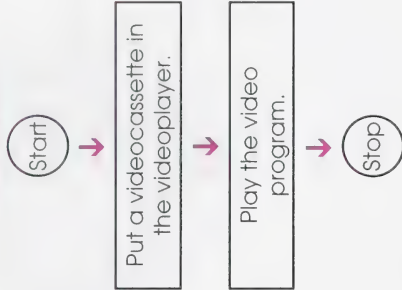


Put a videocassette in a video player and press the play button.

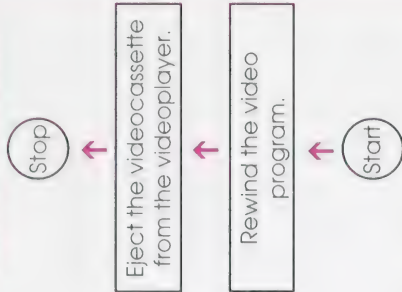
Solution

First write the actions in a flow chart. Then make a reverse flow chart.

Flow Chart



Reverse Flow Chart



From the reverse flow chart, it is clear that you must rewind the video program and eject the videocassette from the videoplayer.



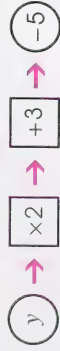
Example 2

Solve the equation $2y + 3 = -5$. Verify the solution.

Solution

First write the equation in a flow chart. (Remember the rules for the order of operations when writing the flow chart.) Then make a reverse flow chart. Use the reverse flow chart to solve the equation.

Flow Chart



Reverse Flow Chart



From the reverse flow chart, it is clear that $y = -4$.

Verification

LS	RS
$2y + 3$	-5
$= 2(-4) + 3$	
$= -8 + 3$	
$= -5$	
LS	$= RS$

2. Use flow charts and reverse flow charts to solve each of the following equations. Verify each solution.

a. $3a - 2 = 7$

b. $2b + 1 = -9$

c. $\frac{c}{3} - 4 = 5$

d. $\frac{d}{2} + 1 = 5$



Check your answers by turning to the Appendix.

Enrichment

In this Enrichment you will translate problems into equations that involve brackets. Because you have not yet learned how to solve this type of equation algebraically, you will use spreadsheets.

Example



Telephone companies have different long distance savings plans.

One savings plan allows a customer to select a community (within a 100-km radius) that the customer would like to call. Under the plan the telephone company charges \$7 for the first 60 min of calls to the selected community and \$0.15 for each additional minute of calls.

Gena has subscribed to this selected route plan. Last month Gena was charged \$9.55 on this plan. How long did she talk to people in the selected community?

Solution

Step 1: Make up a chart showing the time talked and the cost. Then establish a pattern for the cost of calls over 60 min to the selected community. **Note:** There is a flat charge of \$7 for the first 60 min of calls.

Time Talked (min)	Cost (\$)
60	7.00
61	7.15
62	7.30
63	7.45
t	$7.00 + 0.15(t - 60)$

$$\begin{aligned} \rightarrow c &= 7.00 + 0.15(61 - 60) \\ \rightarrow c &= 7.00 + 0.15(62 - 60) \\ \rightarrow c &= 7.00 + 0.15(63 - 60) \end{aligned}$$

Step 2: Write a formula.

Let c be the cost (in dollars) and t be the time (in min) over 60 min.

$$c = 7.00 + 0.15(t - 60)$$

Step 3: Start a new spreadsheet document and enter the following data and formulas to calculate the cost of a sequence of calls lasting 60 min or longer.

Note: The formula in cell A3 increases the number of minutes talked by 1. The formula in cell B3 corresponds to the algebraic formula for the cost of the call.

	A	B
1	Minutes Talked	Cost
2	60	7
3	$= A2 + 1$	$= 7.00 + 0.15 * (A3 - 60)$

Step 4: Fill the cells in columns A and B by doing the following:

- Highlight cell A3 and then drag down the column, or click on cell A3 and shift-click on cell A20. Next, click on "Calculate" in the menu bar and select "Fill Down."
- Highlight cell B3 and then drag down the column, or click on cell B3 and shift-click on cell B20. Next, click on "Calculate" in the menu bar and select "Fill Down."

Step 5: To solve the problem, find \$9.55 in column B and then read the corresponding number in column A.

A		B
1	Minutes Talked	Cost
2	60	7
3	61	7.15
4	62	7.3
5	63	7.45
6	64	7.6
7	65	7.75
8	66	7.9
9	67	8.05
10	68	8.2
11	69	8.35
12	70	8.5
13	71	8.65
14	72	8.8
15	73	8.95
16	74	9.1
17	75	9.25
18	76	9.4
19	77	9.55
20	78	9.7

Gena talked for 77 min.

← Examine Row 19.



Start new documents in a spreadsheet program to answer questions 1 and 2.

1.



Jonathon is calling his friend who lives in Greece. The telephone company charges \$2.48 for the first minute and \$1.65 for each additional minute.

If Jonathon has only \$20 for the call, what is the maximum length of time he can spend on the phone?

2.

A pizzeria charges \$7.95 for a medium pizza with two toppings. Additional toppings cost \$0.85 each. (The prices include GST.)

Jocelyn and her friends want to order a pizza. They have only \$12.00. What is the maximum number of pizza toppings that the friends can order?



Check your answers by turning to the Appendix.

Conclusion



In this section you solved equations and verified the solutions using models and a paper-and-pencil method. You also used equation-solving skills to solve problems and evaluate formulas.

You found that maintaining balance is very important in solving equations. For this reason, a balance scale is used to visualize equations. The scale is a reminder that whenever you perform an operation on one side of an equation, you must perform the same operation on the other side of the equation.

Learning to solve equations is like learning to ride a bicycle. Maintaining balance becomes easier with practice.

Assignment



You are now ready to complete the assignment for Section 2.

Module Summary



THE EDMONTON JOURNAL

In this module you investigated a branch of mathematics called algebra. You became familiar with the language of algebra and you translated English phrases into math and vice versa. To help you visualize algebraic expressions and equations, you used learning aids to model them. You solved equations by adding, subtracting, multiplying, and dividing. You used equations to solve problems. You worked with formulas. You discovered how formulas are used in spreadsheets.

Mathematics is a universal language and the meanings of algebraic expressions and equations are understood by mathematicians all over the world.

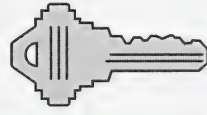
The language of mathematics can be compared to the language of mimes. Mathematicians use variables, numerals, and operation signs to describe patterns and relations. Mimes use facial expressions, gestures, and body language to tell a story.

Final Module Assignment



You are now ready to complete the final module assignment for Module 2.

APPENDIX



Glossary

Suggested Answers

Articles

Cut-out Learning Aids

Glossary

Algebra: the branch of mathematics that uses variables to study the relationship between numbers

Algebraic expression: a combination of variables, numbers, and arithmetic operations; also called a **variable expression**

Conditional equation: an equation that is only true for certain values of the variable

Formula: an equation that shows how one quantity is related to one or more other quantities

Problem: a task for which the method of finding the answer (as well as the answer) is not immediately known

Sequence: a set of numbers or pictures that is ordered according to some rule or formula

Solution (of an equation): the value of the variable that makes the equation true

Spreadsheet: a computer-generated arrangement of data in rows and columns

Technology: the application of tools, materials, and processes to problem solving; more specifically, devices and systems used in processing, transferring, storing, and communicating information through electronic media

Term: each number or item in a sequence

The first number in a sequence is called the first term, the second number is called the second term, and so on.

Variable: a quantity, represented by a symbol that may have many values

Suggested Answers

Section 1: Activity 1

1. a. **Step 1:** Write the year without the century number and label it a .

$$a = 69$$

- Step 2:** Divide a by 4. Then write the quotient and label it b .
(Ignore any remainder.)

$$\begin{array}{r} 17 \text{ ← Quotient} \\ 4 \overline{)69} \\ \underline{4} \\ 29 \\ \underline{28} \\ 1 \end{array}$$

$b = 17$

- Step 3:** Use the given chart to write the number for the month and label it c .

$$c = 6$$

Step 4: Write the number of the day of the month and label it *d*.

$$d = 20$$

Step 5: Add *a*, *b*, *c*, and *d*. Divide this sum by 7. Write the remainder and label it *e*.

$$\begin{array}{r} 16 \\ 7 \overline{)112} \\ 69 \\ 17 \\ 6 \\ + 20 \\ \hline 112 \\ 0 \leftarrow \text{Remainder} \end{array}$$

Step 6: Refer to the given chart to find the day of the week.

July 20, 1969, was a Sunday.

b. Step 1: Write the year without the century number and label it *a*.

$$a = 17$$

Step 2: Divide *a* by 4. Then write the quotient and label it *b*. (Ignore any remainder.)

$$\begin{array}{r} 4 \leftarrow \text{Quotient} \\ 4 \overline{)17} \\ 16 \\ \hline 1 \end{array}$$

$$b = 4$$

Step 3: Use the given chart to write the number for the month and label it *c*.

$$c = 5$$

Step 4: Write the number of the day of the month and label it *d*.

$$d = 6$$

Step 5: Add *a*, *b*, *c*, and *d*. Divide the sum by 7. Write the remainder and label it *e*.

$$\begin{array}{r} 4 \\ 7 \overline{)32} \\ 28 \\ \hline 4 \leftarrow \text{Remainder} \end{array}$$

Step 6: Refer to the given chart to find the day of the week.

December 6, 1917, was a Thursday.

c. Step 1: Write the year without the century and label it *a*.

$$a = 88$$

Step 2: Divide a by 4. Then write the quotient and label it b .
(Ignore any remainder.)

$$\begin{array}{r} 22 \text{ ← Quotient} \\ 4 \overline{)88} \\ \underline{8} \\ 8 \\ \underline{8} \\ 0 \end{array}$$

$b = 22$

Step 3: Use the given chart to write the number for the month and label it c .

$c = 2$

1988 is a leap year.

Step 4: Write the number of the day of the month and label it d .

$d = 13$

Step 5: Add a , b , c , and d . Divide this sum by 7. Write the remainder and label it e .

$$\begin{array}{r} 88 \\ 22 \\ 2 \\ + 13 \\ \hline 125 \end{array}$$

$e = 6$

$$\begin{array}{r} 17 \\ 7 \overline{)125} \\ \underline{7} \\ 55 \\ \underline{49} \\ 6 \text{ ← Remainder} \end{array}$$

Step 6: Refer to the given chart to find the day of the week.

February 13, 1988, was a Saturday.

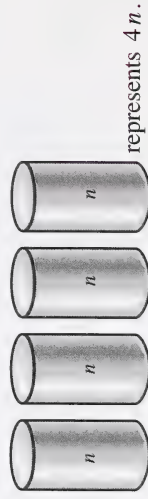
2. Answers will vary. Following are acceptable answers.

- four times n , the product of four and n ; n multiplied by four
- n minus three; n decreased by three; three less than n ; n take away three; three subtracted from n
- n plus two; n increased by two; two more than n ; the sum of n and two; two added to n
- six minus n ; six take away n ; six decreased by n ; n subtracted from six

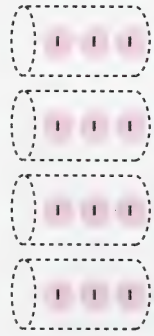
3. a. If $n = -3$, then

$$\begin{aligned} &4n \\ &= 4(-3) \\ &= -12 \end{aligned}$$

Following is an explanation of how learning aids can be used to illustrate the answer.



If $n = -3$, replace each cylinder with three negative counters.

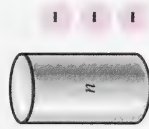


The result is 12 negative counters. This represents -12 .

b. If $n = -3$, then

$$\begin{aligned} n - 3 \\ &= -3 - 3 \\ &= -3 + (-3) \\ &= -6 \end{aligned}$$

Following is an explanation of how learning aids can be used to illustrate the answer.



represents $n - 3$.

If $n = -3$, replace the cylinder with three negative counters.

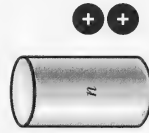


The result is six negative counters. This represents -6 .

c. If $n = -3$, then

$$\begin{aligned} n + 2 \\ &= -3 + 2 \\ &= -1 \end{aligned}$$

Following is an explanation of how learning aids can be used to illustrate the answer.



represents $n + 2$.

If $n = -3$, replace the cylinder with three negative counters.



Simplify by removing any zero pairs.

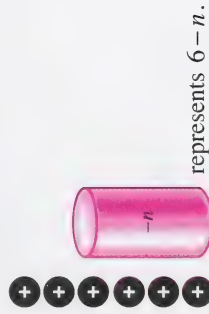


The result is one negative counter. This represents -1 .

d. If $n = 3$, then

$$\begin{aligned} 6 - n \\ &= 6 - (-3) \\ &= 6 + (+3) \\ &= 9 \end{aligned}$$

Following is an explanation of how learning aids can be used to illustrate the answer.



represents $6 - n$.

If $n = -3$, replace the cylinder with three positive counters.
Note: The value of $-n$ is the opposite of n .



The result is nine positive counters. This represents $+9$.

4. Answers will vary. Following are acceptable answers.

- five less than two times x ;
two times x , minus five
- two times, five less than x ;
two times, x minus five

5. a. If $x = 3$, then

$$\begin{aligned} 2x - 5 \\ &= 2(3) - 5 \\ &= 6 - 5 \\ &= 1 \end{aligned}$$

Following is an explanation of how learning aids can be used to illustrate the answer.



represents $2x - 5$.

If $x = 3$, replace each cylinder with three positive counters.



Simplify by removing any zero pairs.



The result is one positive counter. This represents $+1$.

b. If $x = 3$, then

$$\begin{aligned} & 2(x - 5) \\ &= 2(3 - 5) \\ &= 2[3 + (-5)] \\ &= 2(-2) \\ &= -4 \end{aligned}$$

Following is an explanation of how learning aids can be used to illustrate the answer.



represents $2(x - 5)$.

If $x = 3$, replace each cylinder with three positive counters.



Simplify by removing any zero pairs.



The result is four negative counters. This represents -4 .

6. Variables may vary. Each of the following answers uses the first letter of a word for the variable.

- | | | |
|---------------|------------------|-----------------------|
| a. $a + 2$ | b. $2d$ | c. $5p$ |
| d. $a + 12$ | e. $\frac{e}{2}$ | f. $\frac{2s}{3}$ |
| g. $3(a + j)$ | h. $w - e$ | i. $\frac{r}{2} + 2m$ |

Now Try This

7. You can use a diagram or a number line to help you solve the problem.



Alexa's kite was 140 m above the ground.

8. a. Answers will vary. This is one example. Graham was born in September. He got the following results.

SEPTEMBER						
S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

- i. The sum of the four centre numbers

$$9 + 10 + 16 + 17 = 52$$

- ii. The sum of the four numbers in each diagonal

$$1 + 9 + 17 + 25 = 52$$

$$22 + 16 + 10 + 4 = 52$$

- iii. The sum of the four corner numbers

$$1 + 4 + 22 + 25 = 52$$

- iv. Each set of four numbers equals 52, the number of weeks in a year.

- b. The results are the same as in question 8.a.

- c. The results are the same as in question 8.a.

Section 1: Activity 2

1. a. $c = 12.00 + 0.50t$

$$= 12.00 + 0.50(1)$$

$$= 12.00 + 0.50$$

$$= 12.50$$

It costs \$12.50 to rent the skis for 1 h.

- b. $c = 12.00 + 0.50t$

$$= 12.00 + 0.50(2)$$

$$= 12.00 + 1.00$$

$$= 13.00$$

It costs \$13.00 to rent the skis for 2 h.

- c. $c = 12.00 + 0.50t$

$$= 12.00 + 0.50(3)$$

$$= 12.00 + 1.50$$

$$= 13.50$$

It costs \$13.50 to rent the skis for 3 h.

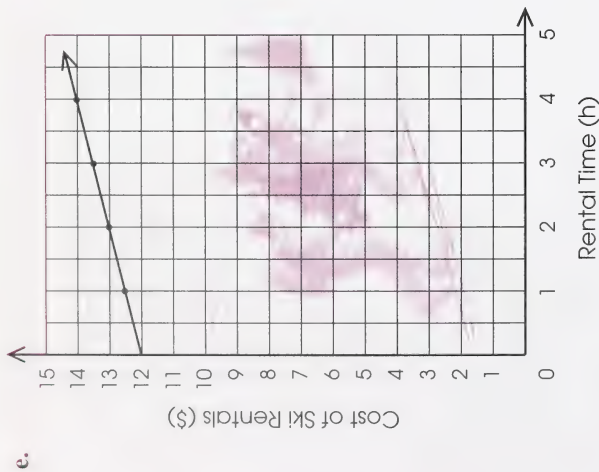
- d. $c = 12.00 + 0.50t$

$$= 12.00 + 0.50(4)$$

$$= 12.00 + 2.00$$

$$= 14.00$$

It costs \$14.00 to rent the skis for 4 h.



f. As the time increases, the cost of renting skis increases.

2. a. $t = \frac{72}{r}$
 $= \frac{72}{4}$
 $= 18$

If the interest rate is 4%, it will take 18 years for an investment to double.

b. $t = \frac{72}{r}$
 $= \frac{72}{8}$
 $= 9$

If the interest rate is 8%, it will take 9 years for an investment to double.

c. $t = \frac{72}{r}$
 $= \frac{72}{12}$
 $= 6$

If the interest rate is 12%, it will take 6 years for an investment to double.

d. $t = \frac{72}{r}$
 $= \frac{72}{16}$
 $= 4.5$

If the interest rate is 16%, it will take 4.5 years for an investment to double.

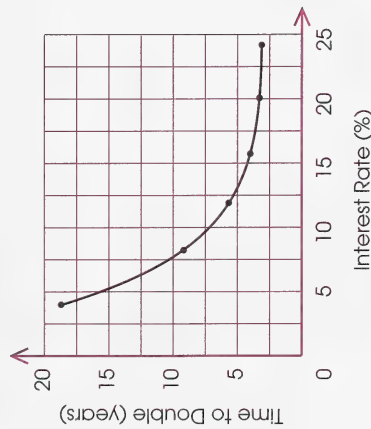
e. $t = \frac{72}{r}$
 $= \frac{72}{20}$
 $= 3.6$

If the interest rate is 20%, it will take 3.6 years for an investment to double.

f. $t = \frac{72}{r}$
 $= \frac{72}{24}$
 $= 3$

If the interest rate is 24%, it will take 3 years for an investment to double.

g. The graph should look similar to the following.



h. As the interest rate increases, the time it takes an investment to double decreases.

3. Before answering a., b., and c., find the reward for each option.

Step 1: Let the unknown quantities be represented by variables.

Let r be the reward (in dollars).

Let n be the number of correct answers.

Step 2: Write a formula for Reward Option 1.

$$r = 4n - 5$$

Step 3: Write a formula for Reward Option 2.

$$r = 3n$$

Step 4: Write a formula for Reward Option 3.

$$\begin{aligned} r &= 10 + 1n \\ &= 10 + n \end{aligned}$$

$1n = n$
 The 1 (one)
 is generally
 not written.

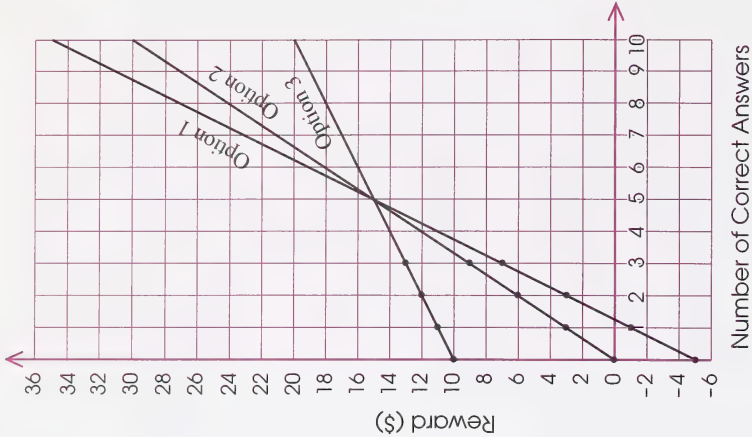
Step 5: Make a table of values for each option.

Reward Option 1	
Number of Correct Answers	Reward (\$)
0	-5
1	-1
2	3
3	7
...	...

Reward Option 2	
Number of Correct Answers	Reward (\$)
0	0
1	3
2	6
3	9
...	...

Reward Option 3	
Number of Correct Answers	Reward (\$)
0	10
1	11
2	12
3	13
...	...

Step 6: Make a graph of Reward Options 1, 2, and 3.



There are no arrowheads on the line because the highest possible number of correct answers is 10.

- a. Nicholas should pick Option 3 if he expects to get no correct answers because he would get a reward of \$10. If he chose Option 2, he would get \$0. If he chose Option 1, he would owe his mother \$5.
- b. Nicholas should pick Option 1 if he expects to get all ten questions correct because he would get a reward of \$35. If he chose Option 2, he would get \$30. If he chose Option 3, he would get \$20.
- c. All the options pay the same amount if Nicholas gets five correct answers. Each pays \$15.

Mass of Roast (kg)	Cooking Time (min)
1	40
2	80
3	120
4	160
5	200

4. a.

b. $c = 40m$

c. $c = 40m$

$$= 40(4.5)$$

$$= 180$$

It will take 180 min or 3 h to cook a 4.5-kg roast.

5. a. Noel did not put an arrowhead on the graph because the plant will not continue to grow forever.

b.

Time (days)	Height (cm)
0	4
1	5
2	6
3	7
4	8
5	9

c. $h = t + 4$

6. a. The graph should look like the following:



b. $d = \frac{5s}{6}$

$$\text{c. } d = \frac{5s}{6}$$

$$= \frac{5(96)}{6}$$

$$= 80$$

The person will travel 80 m while “blind.”

Now Try This

7. a. **Step 1:** Calculate the length of time the light stayed green before turning amber.

$$\begin{array}{r} 09:01:34 \\ - 09:01:10 \\ \hline 24 \end{array}$$

The light stayed green for 24 s before turning amber.

- Step 2:** Calculate the next time the light turned amber.

$$\begin{array}{r} 09:04:10 \\ + 24 \\ \hline 09:04:34 \end{array}$$

The light next turned amber at 09:04:34.

- b. **Step 1:** Calculate the length of time the light stayed amber before turning red.

$$\begin{array}{r} 09:01:37 \\ - 09:01:34 \\ \hline 03 \end{array}$$

The light stayed amber for 3 s before turning red.

- Step 2:** Calculate the next time the light turned red.

$$\begin{array}{r} 09:04:34 \\ + 3 \\ \hline 09:04:37 \end{array}$$

The light next turned red at 09:04:37.

- c. **Step 1:** Calculate the length of time the light stayed red.

$$\begin{array}{r} 09:02:10 \\ - 09:01:37 \\ \hline 33 \end{array}$$

The light stayed red for 33 s.

- Step 2:** Calculate the next time the light turned green.

$$\begin{array}{r} 09:04:37 \\ + 33 \\ \hline 09:04:70 = 09:05:10 \end{array}$$

The light next turned green at 09:05:10.

- d. You can use patterns to solve this problem.

Colour	Time	Pattern
Green	09:01:10	
Amber	09:01:34	
Red	09:01:37	
...	...	
Green	09:04:10	+24
Amber	09:04:34	+3
Red	09:04:37	+33
Green	09:05:10	+24
Amber	09:05:34	+3
Red	09:05:37	+33
Green	09:06:10	+24
Amber	09:06:34	

The light was amber at 09:06:34.

- e. There was probably more northbound/southbound traffic than there was eastbound/westbound traffic.

8. You can use diagrams to help you solve the problem. **Note:** In the following diagram (W) indicates that the elevator waits to take on passengers.

Floor	Faster	Slower
7	10:00:00	10:00:00
6	10:00:01 (W)	10:00:02
5	10:00:05 (W)	10:00:04 (W)
4	10:00:06 (W)	10:00:09
3	10:00:10 (W)	10:00:11
2	10:00:14 (W)	10:00:13 (W)
1	10:00:15	10:00:17

Mark will reach the first floor at 10:00:15. Marshall will reach the first floor at 10:00:17.

Section 1: Activity 3

1. **Step 1:** Find a pattern.

1950, 1962, 1974, 1986, ...
 +12 +12 +12

Step 2: Find the next three years.

1950, 1962, 1974, 1986, 1998, 2010, 2022

The next three years of the Tiger are 1998, 2010, and 2022.

2. a. **Step 1:** Find a pattern in the sequence.

6, 11, 16, 21, 26, ...
+5 +5 +5 +5

Step 2: Use patterns and the guess, check, and revise strategy to find a formula that tells how a term (t) is related to its position (n) in the sequence.

Position of Term (n)	Term (t)
1st	6
2nd	11
3rd	16
4th	21
5th	26
...	...
n th	$5n + 1$

The formula is $t = 5n + 1$.

Step 3: Use the formula to predict the 30th term.

$$\begin{aligned}
 t &= 5n + 1 \\
 &= 5(30) + 1 \\
 &= 150 + 1 \\
 &= 151
 \end{aligned}$$

The 30th term is 151.

b. **Step 1:** Find a pattern in the sequence.

3, 7, 11, 15, 19, ...
+4 +4 +4 +4

Step 2: Use patterns and the guess, check, and revise strategy to find a formula that tells how a term (t) is related to its position (n) in the sequence.

Position of Term (n)	Term (t)
1st	3
2nd	7
3rd	11
4th	15
5th	19
...	...
n th	$4n - 1$

The formula is $t = 4n - 1$.

Step 3: Use the formula to predict the 30th term.

$$\begin{aligned} t &= 4n - 1 \\ &= 4(30) - 1 \\ &= 120 - 1 \\ &= 119 \\ t &= 0.5n + 1 \\ &= 0.5(30) + 1 \\ &= 15 + 1 \\ &= 16 \end{aligned}$$

The 30th term is 119.

c. Step 1: Find a pattern in the sequence.

$$1.5, 2, 2.5, 3, 3.5, \dots$$

Step 2: Use patterns and the guess, check, and revise strategy to find a formula that tells how a term (t) is related to its position (n) in the sequence.

Position of Term (n)	Term (t)
1st	1.5
2nd	2
3rd	2.5
4th	3
5th	3.5
...	...
n th	$0.5n + 1$

$\downarrow$ $t = 0.5 \times 1 + 1$
 $\downarrow$ $t = 0.5 \times 2 + 1$
 $\downarrow$ $t = 0.5 \times 3 + 1$
 $\downarrow$ $t = 0.5 \times 4 + 1$
 $\downarrow$ $t = 0.5 \times 5 + 1$
 $\downarrow$ $t = 0.5n + 1$

The formula is $t = 0.5n + 1$.



3. Step 1: Find a pattern in the sequence.

Step 3: Use the formula to predict the 30th term.

$$\begin{aligned} t &= 0.5n + 1 \\ &= 0.5(30) + 1 \\ &= 15 + 1 \\ &= 16 \end{aligned}$$

The 30th term is 16.

Step 2: Use patterns and the guess, check, and revise strategy to find a formula relating the number of circles (c) in a picture to its position (n).

Position of Term (n)	Number of Circles (c)
1st	5
2nd	8
3rd	11
...	...
n th	$3n + 2$

$\downarrow$ — $c = 3 \times 1 + 2$
 $\downarrow$ — $c = 3 \times 2 + 2$
 $\downarrow$ — $c = 3 \times 3 + 2$
 $\downarrow$ — $c = 3n + 2$

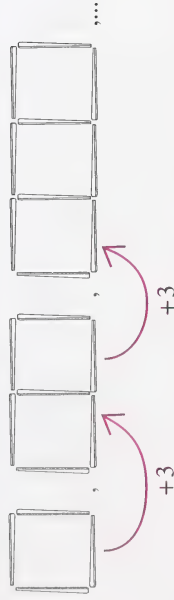
The formula is $c = 3n + 2$.

Step 3: Use the formula to predict the number of circles in the 40th picture.

$$\begin{aligned}
 c &= 3n + 2 \\
 &= 3(40) + 2 \\
 &= 120 + 2 \\
 &= 122
 \end{aligned}$$

There will be 122 circles in the 40th picture.

4. **Step 1:** Find a pattern in the sequence.



Step 2: Use patterns and the guess, check, and revise strategy to find a formula relating the number of toothpicks (t) in a picture to its position (n).

Position of Term (n)	Number of Toothpicks (t)
1st	4
2nd	7
3rd	10
...	...
n th	$3n + 1$

$\downarrow$ — $t = 3 \times 1 + 1$
 $\downarrow$ — $t = 3 \times 2 + 1$
 $\downarrow$ — $t = 3 \times 3 + 1$
 $\downarrow$ — $t = 3n + 1$

The formula is $t = 3n + 1$.

Step 3: Use the formula to predict the number of toothpicks in the 30th picture.

$$\begin{aligned}
 t &= 3n + 1 \\
 &= 3(30) + 1 \\
 &= 90 + 1 \\
 &= 91
 \end{aligned}$$

There will be 91 toothpicks in the 30th picture.

5. **Step 1:** Use patterns and the guess, check, and revise strategy to find a formula relating the number of dots (d) to the number of squares (s).

Number of Squares (s)	Number of Dots (d)
5	4
7	6
9	8
s	$s - 1$

$$\begin{aligned} \rightarrow d &= 5 - 1 \\ \rightarrow d &= 7 - 1 \\ \rightarrow d &= 9 - 1 \\ \rightarrow d &= s - 1 \end{aligned}$$

The formula is $d = s - 1$.

- Step 2:** Use the formula to predict the number of dots in the picture with 15 squares.

$$\begin{aligned} d &= s - 1 \\ &= 15 - 1 \\ &= 14 \end{aligned}$$

There will be 14 dots in the picture.

6. **Step 1:** Use patterns and the guess, check, and revise strategy to find a formula relating the number of circles (c) to the number of triangles (t).

Number of Triangles (t)	Number of Circles (c)
2	2
3	4
4	6
$\dots$	$\dots$
t	$2t - 2$

$$\begin{aligned} \rightarrow c &= 2 \times 2 - 2 \\ \rightarrow c &= 2 \times 3 - 2 \\ \rightarrow c &= 2 \times 4 - 2 \\ \rightarrow c &= 2t - 2 \end{aligned}$$

The formula is $c = 2t - 2$.

- Step 2:** Use the formula to predict the number of circles in the picture with 15 triangles.

$$\begin{aligned} c &= 2t - 2 \\ &= 2(15) - 2 \\ &= 30 - 2 \\ &= 28 \end{aligned}$$

There will be 28 circles in the picture.

7. **Step 1:** Use patterns and the guess, check, and revise strategy to find the formula relating the number of crosspieces (c) to the number of posts (p).

Number of Posts (p)	Number of Crosspieces (c)
2	4
3	8
4	12
...	...
p	$4p - 4$

$$\begin{aligned} &\rightarrow c = 4 \times 2 - 4 \\ &\rightarrow c = 4 \times 3 - 4 \\ &\rightarrow c = 4 \times 4 - 4 \\ &\rightarrow c = 4p - 4 \end{aligned}$$

The formula is $c = 4p - 4$.

- Step 2:** Use the formula to predict the number of crosspieces in the fence with 32 posts.

$$\begin{aligned} c &= 4p - 4 \\ &= 4(32) - 4 \\ &= 128 - 4 \\ &= 124 \end{aligned}$$

Harold will need 124 crosspieces.

8. **Step 1:** Use patterns and the guess, check, and revise strategy to find the formula relating the number of boxes in a layer (b) to the position of the layer (n).

Position of Layer (n)	Number of Boxes in Layer (b)
1st	1
2nd	3
3rd	5
...	...
n	$2n - 1$

$$\begin{aligned} &\rightarrow b = 2 \times 1 - 1 \\ &\rightarrow b = 2 \times 2 - 1 \\ &\rightarrow b = 2 \times 3 - 1 \\ &\rightarrow b = 2n - 1 \end{aligned}$$

The formula is $b = 2n - 1$.

- Step 2:** Use the formula to predict the number of boxes in the 10th layer.

$$\begin{aligned} b &= 2n - 1 \\ &= 2(10) - 1 \\ &= 20 - 1 \\ &= 19 \end{aligned}$$

There will be 19 boxes in the 10th layer.

Now Try This

9. Use the guess, check, and revise strategy to answer a. and b. Other answers are possible.

a. $9 + 9 \div 9 + 9 - 9$ b. $4 \times 4 + 4 \div 4$

Section 1: Activity 4

1. Your spreadsheet will look like this.

	A	B
1	Park	Area (ha)
2	Banff	664080
3	Yoho	131310
4	Glacier	134940
5	Waterton	52590
6	Jasper	108780

2. You will enter the following words and formulas.

	A	B
1	Park	Area (ha)
2	Banff	664080
3	Yoho	131310
4	Glacier	134940
5	Waterton	52590
6	Jasper	108780
7	Total Area	=B2+B3+B4+B5+B6
8	Average Area	=B7/5

Your spreadsheet will look like this. The calculations for cells B7 and B8 are done by the computer. **Note:** If you click on either cell B7 or cell B8, you will notice the formula in the entry bar.

	A	B
1	Park	Area (ha)
2	Banff	664080
3	Yoho	131310
4	Glacier	134940
5	Waterton	52590
6	Jasper	108780
7	Total Area	1091700
8	Average Area	218340

- a. The total area of the five oldest national parks is 1 091 700 ha.
- b. The average area of each park is 218 340 ha.
3. a. The total in cell B8 is the total number of cars.
The total in cell C8 is the total number of trucks.
The total in cell D8 is the total number of vehicles (cars and trucks).
The total in cell D2 is the total number of white vehicles (cars and trucks).
The total in cell D3 is the total number of black vehicles.
The total in cell D4 is the total number of red vehicles.
The total in cell D5 is the total number of blue vehicles.
The total in cell D6 is the total number of green vehicles.
The total in cell D7 is the total number of vehicles of other colours.

- b. To get the average in cell B9, enter

$$= (B2 + B3 + B4 + B5 + B6 + B7) / 6$$
 or
$$= B8 / 6$$
.
- To get the average in cell C9, enter

$$= (C2 + C3 + C4 + C5 + C6 + C7) / 6$$
 or
$$= C8 / 6$$
.
- To get the average in cell D9, enter

$$= (D2 + D3 + D4 + D5 + D6 + D7) / 6$$
 or
$$= D8 / 6$$
.
- c. To get the total in cell D8, enter

$$= D2 + D3 + D4 + D5 + D6 + D7$$
 or
$$= B8 + C8$$
.
- d. i. cell D5 ii. cell C9 iii. cell B8
4. a. Formulas in spreadsheets are similar to algebraic formulas in that they provide directions for how to calculate a quantity. They also relate one quantity to another quantity.
- b. Formulas in spreadsheets are different from algebraic formulas in that cell addresses are used instead of variables. Also, only the equal sign and the right side of the equation is given.
5. a. i. $= 50 * A2 / 68$ ii. $= 50 * A3 / 68$
 iii. $= 50 * A4 / 68$ iv. $= 50 * A5 / 68$
- b. Answers will vary. If you are using *ClarisWorks™*, you should do the following steps to change the format of the numbers in Column B.

Step 1: Select a range of cells by clicking on cell A2 and shift-clicking on cell A5.

Step 2: Choose “Number” from the Format menu. A Numeric dialogue box like the one shown here will appear.

Specify a fixed decimal point by selecting “Fixed.”

Specify a precision for the format by entering 0 (for no decimal places) in the box. **Note:** *ClarisWorks™* is preset to two decimal places for Fixed, Currency, and Percent formats.

Then click on “OK.”

The numbers in column B will be rounded to the nearest whole number.

- c. Answers will vary. If you are using *ClarisWorks™*, you may create a graph of the relationship by following these steps.

Step 1: Select a range of cells by clicking on the first cell (A1) and dragging through the cells until all the cells are highlighted. You may also click on cell A1 and then shift-click on cell B5.

	A	B	C
1	Child's Mass (kg)	Child's Dose (mg)	
2	10	7.35294117647058	
3	20	14.7058823529411	
4	30	22.0588235294117	
5	40	29.4117647058823	
6			

Step 2: Choose "Make Chart" from the Options menu. A Chart Options dialogue box like the one shown here will appear.

Modify

Gallery

Axis

Series

Labels

General

Chart Options

Bar

Area

Line

Scatter

X-Y

Stacked Bar

Stacked Area

Stacked Line

Pie

Scatter

X-Y

Stacked

Hi-Low

Pictogram

Stacked Pictogram

Color

Horizontal

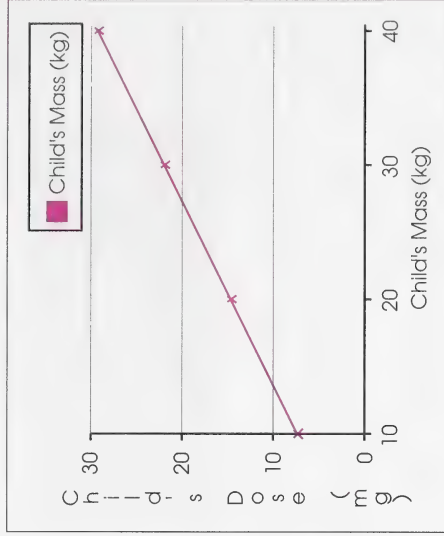
Shadow

3-dimensional

Cancel

OK

Select x-y Line from the dialogue box. Press "Return" or click on the OK button. The computer makes a graph like this.



Note: You may modify your graph by changing the series symbol.

Did You Know?

- These varieties of wheat ripened earlier and were resistant to disease. These characteristics made the wheat successful in Canada.

7. $0.18 \times 293\,000 = 52\,740$

About 52 740 farms in Canada are classified as wheat farms.

Now Try This

8. You can use logical reasoning and work backwards to solve this problem. You can also use play money and act out the problem.

Step 1: Make a flow chart to show the order of the actions. Then make a reverse flow chart.

Flow Chart



Reverse Flow Chart



Step 2: Use the reverse flow chart to solve the problem.

Mr. Sabiston started with \$140.

Section 1: Follow-up Activities

Extra Help

1. **Step 1:** Put the given data in a table.

Number of Filing Trays	Number of Riser Rods
2	4
3	8
4	12

Step 2: Make a guess about how the number of riser rods is related to the number of filing trays. Test the guess, and revise the guess if necessary. Continue guessing, testing, and revising until you find the relationship.

The formula is $r = 4f - 4$, where r is the number of riser rods and f is the number of filing trays.

2. **Step 1:** Put the given data in a table.

Number of Tables	Number of People Seated
2	6
3	8
4	10

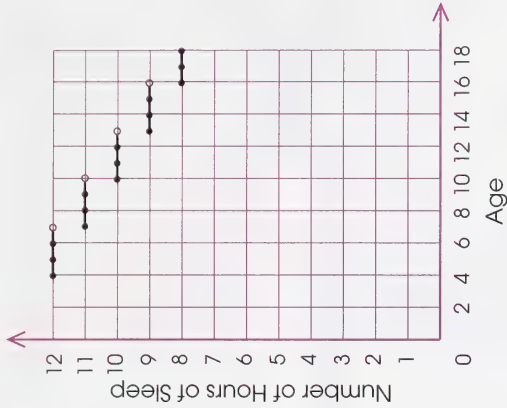
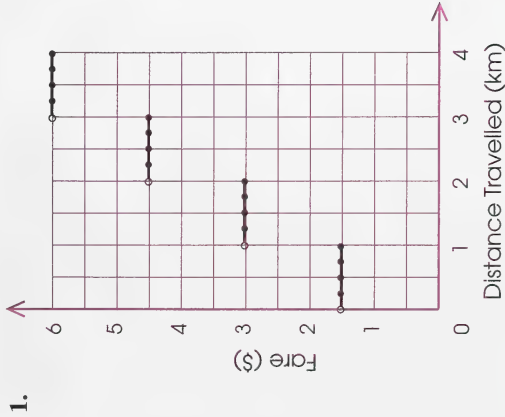
Step 2: Make a guess about how the number of people that can be seated is related to the number of tables. Test the guess, and revise the guess if necessary. Continue guessing, testing, and revising until you find the relationship.

The formula is $p = 2t + 2$, where p is the number of people who can be seated and t is the number of tables.

3. Use the guess, check, and revise strategy to find a formula.

The formula is $e = 1.25n - 0.50$, where e is the earnings (in \$) and n is the number of cars washed.

Enrichment



Section 2: Activity 1

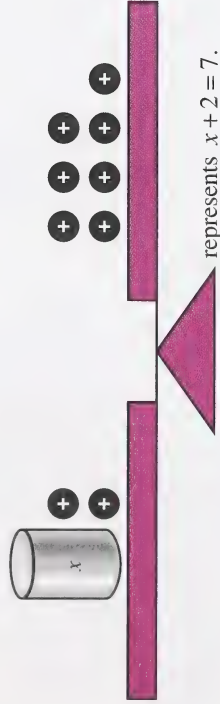
- The capital of Canada is Ottawa .
 - The boiling point of water is 100 °C.
 - The freezing point of water is 0 °C
 - The hummingbird is the smallest bird.
- $5 + x = 13$
 - $5 + 4 = x$
 - $8 - 3 = x + 4$
 - $4 + 1 = x - 2$
- $5 + 8 = 13$
 - $5 + 4 = 9$
 - $8 - 3 = 1 + 4$
 - $4 + 1 = 7 - 2$

4. Check your answers by viewing the video.

5. **a.** Subtract 2 or add -2 to each side. **b.** Add 5 to each side.
c. Subtract 3 or add -3 to each side. **d.** Add 2 to each side.
e. Divide each side by 3. **f.** Divide each side by -2 .

$$\begin{array}{r} \text{6. a. } x + 2 = 7 \\ \quad \underline{-2 \quad -2} \\ x = 5 \end{array}$$

Following is an explanation of how learning aids can be used to illustrate the answer.



Isolate the cylinder. Add two negative counters to **each** side of the equation.



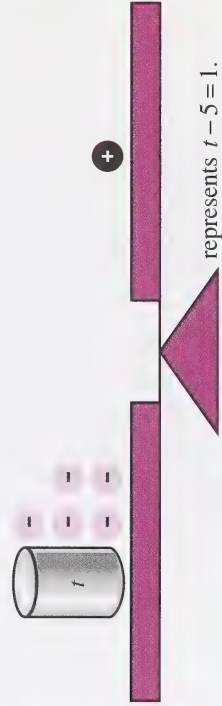
Simplify by removing any zero pairs. This is the result.



The model illustrates that $x = 5$.

$$\begin{array}{r} \text{b. } t - 5 = 1 \\ \quad \underline{+5 \quad +5} \\ t = 6 \end{array}$$

Following is an explanation of how learning aids can be used to illustrate the answer.



Isolate the cylinder. Add five positive counters to **each** side of the equation.



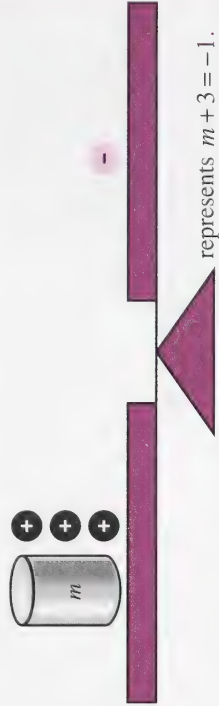
Simplify by removing any zero pairs. This is the result.



The model illustrates that $t = +6$.

$$\begin{array}{r} \text{c. } m + 3 = -1 \\ \quad \quad \quad \cancel{-3} \quad \quad \quad -3 \\ \hline m = -4 \end{array}$$

Following is an explanation of how learning aids can be used to illustrate the answer.



represents $m + 3 = -1$.

Isolate the cylinder. Add three negative counters to **each** side.



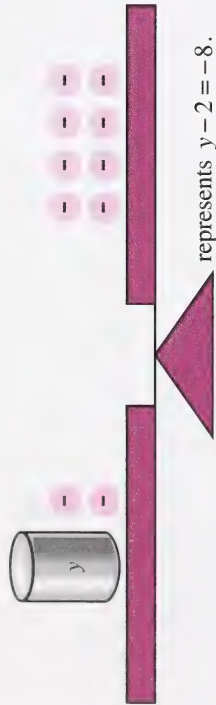
Simplify by removing any zero pairs. This is the result.



The model illustrates that $m = -4$.

d. $y - 2 = -8$
 ~~$+2$~~ $+2$
 $y = -6$

Following is an explanation of how learning aids can be used to illustrate the answer.



Isolate the cylinder. Add two positive counters to **each** side.



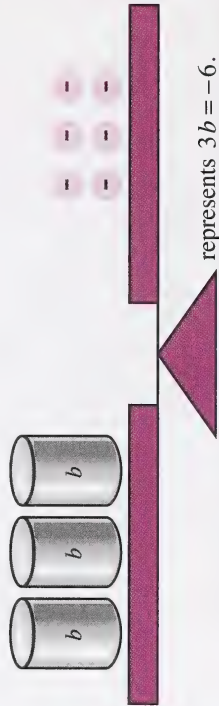
Simplify by removing any zero pairs. This is the result.



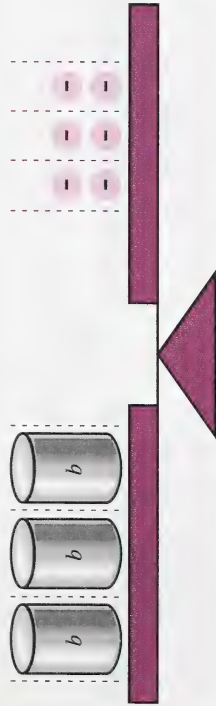
The model illustrates that $y = -6$.

e. $\frac{3b}{3} = \frac{-6}{3}$
 $b = -2$

Following is an explanation of how learning aids can be used to illustrate the answer.



Arrange **each** side into three equal groups.



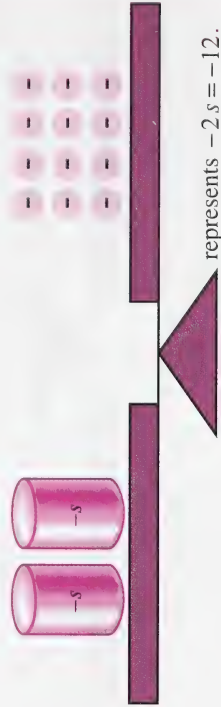
Examine **one** group from **each** side.



The model illustrates that $b = -2$.

f.
$$\frac{-2s}{-2} = \frac{-12}{-2}$$
$$s = +6$$

Following is an explanation of how learning aids can be used to illustrate the answer.



represents $-2s = -12$.

Arrange **each** side into two equal groups.



Examine **one** group from **each** side.



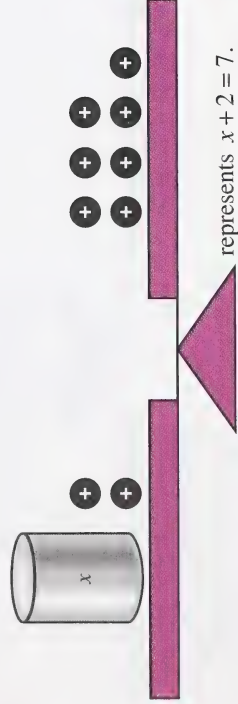
To solve for s , trade the learning aids on each side for their opposites. **Note:** s is the opposite of $-s$; $+6$ is the opposite of -6 .



The model illustrates that $s = 6$.

$$\begin{array}{r|l} \text{LS} & \text{RS} \\ \hline x+2 & 7 \\ =5+2 & \\ =7 & \end{array} \quad \text{LS} = \text{RS}$$

Following is an explanation of how learning aids can be used to illustrate the answer.



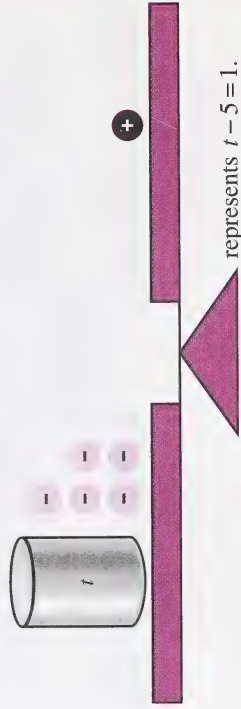
To verify that $x = 5$, replace the cylinder with five positive counters.



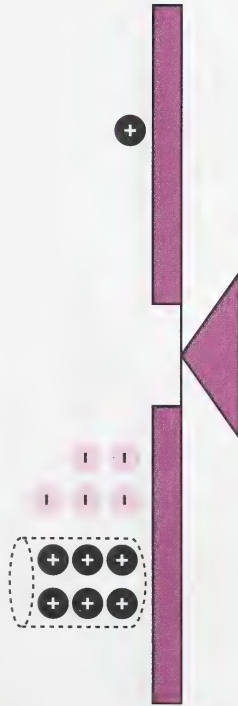
The left side equals the right side.

$$\begin{array}{r|l} \text{LS} & \text{RS} \\ \hline t-5 & 1 \\ =6-5 & \\ =1 & \end{array} \quad \text{LS} = \text{RS}$$

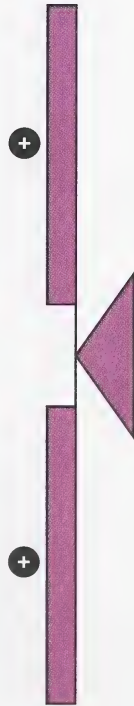
Following is an explanation of how you can use learning aids to illustrate the answer.



To verify that $t = 6$, replace the cylinder with six positive counters.



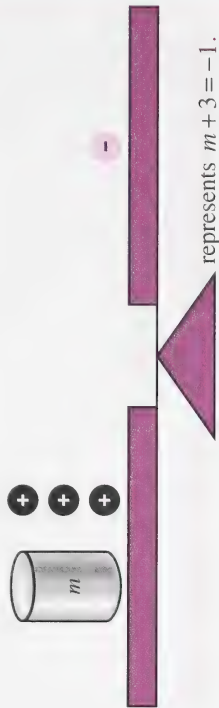
Remove any zero pairs. This is the result.



The left side equals the right side.

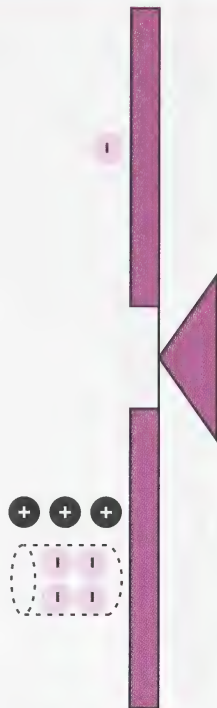
$$\begin{array}{r|l} \text{c.} & \text{LS} \quad \text{RS} \\ & m+3 \quad -1 \\ & = -4+3 \\ & = -1 \\ & \text{LS} = \text{RS} \end{array}$$

Following is an explanation of how learning aids can be used to illustrate the answer.



represents $m + 3 = -1$.

To verify that $m = -4$, replace the cylinder with four negative counters.



Remove any zero pairs. This is the result.



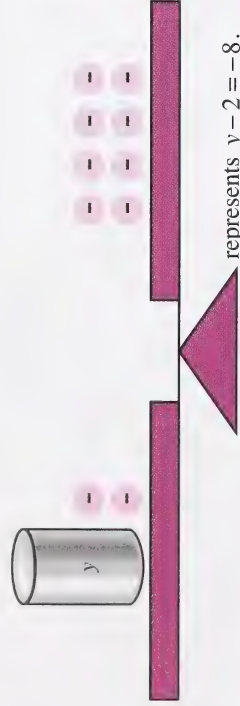
The left side equals the right side.

d.

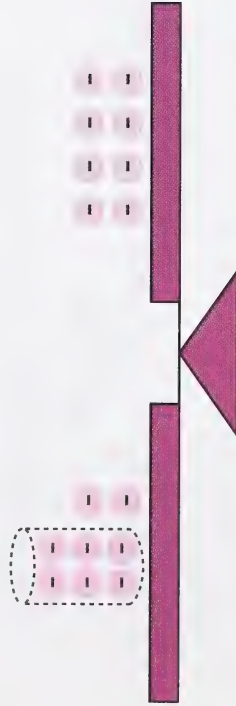
LS	RS
$y - 2$	-8
$= -6 - 2$	
$= -8$	

LS = RS

Following is an explanation of how learning aids can be used to illustrate the answer.



To verify that $y = -6$, replace the cylinder with six negative counters.

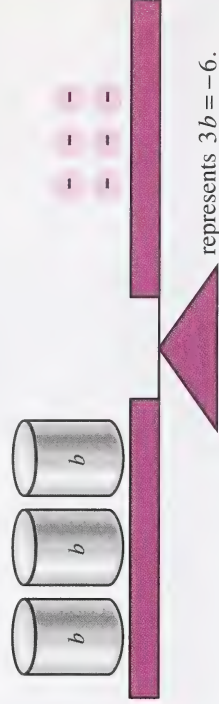


e.

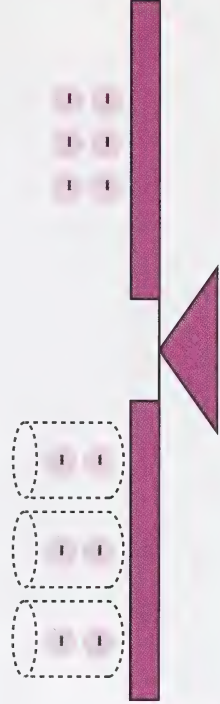
LS	RS
$3b$	-6
$= 3(-2)$	
$= -6$	

LS = RS

Following is an explanation of how learning aids can be used to illustrate the answer.



To verify that $b = -2$, replace **each** cylinder with two negative counters.

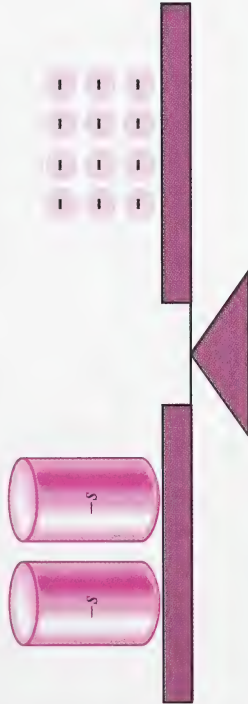


f.

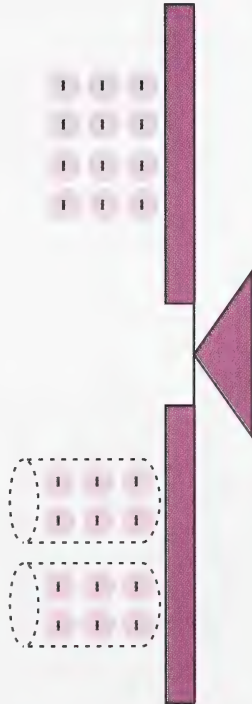
LS	RS
$-2s$	-12
$= -2(6)$	
$= -12$	

LS = RS

Following is an explanation of how learning aids can be used to illustrate the answer.



To verify that $s = 6$ (and $-s = -6$), replace **each** cylinder with six negative counters. **Note:** $-s$ is the opposite of s . If $s = 6$, then $-s = -6$.



The left side equals the right side.

8. a.

$$4\left(\frac{n}{4}\right) = 4(3)$$

$$n = 12$$

b.

$$2\left(\frac{m}{2}\right) = 2(-3)$$

$$m = -6$$

LS	RS
$\frac{n}{4}$	3
$= \frac{12}{4}$	
$= 3$	

LS = RS

LS	RS
$\frac{m}{2}$	-3
$= \frac{-6}{2}$	
$= -3$	

LS = RS

9. **Step 1:** Write an equation to describe the situation.

Let n be the number of tea bags in a box.

$$n + 5 = 125$$

Step 2: Solve the equation.

$$\begin{array}{r} n + 5 = 125 \\ -5 \quad -5 \\ \hline n = 120 \end{array}$$

The box had 120 tea bags.

Note: Determine if the answer is reasonable.

Does 5 tea bags plus 120 tea bags equal 125 tea bags?

Yes. The answer is reasonable.

10. Step 1: Write an equation to describe the situation.

Let n be the number of days in the term.

$$6n = 654$$

Step 2: Solve the equation.

$$\frac{6n}{6} = \frac{654}{6}$$
$$n = 109$$

The term had 109 days.

Note: Determine if the answer is reasonable.

Does 6 sheets of paper per day for 109 days equal 654 sheets of paper?

Yes. The answer is reasonable.

11. Step 1: Write an equation to describe the situation.

Let p be the price (in \$) of the adult ticket.

$$p + 5 = 23$$

Step 2: Solve the equation.

$$\begin{array}{r} p + \cancel{5} = 23 \\ - \cancel{5} -5 \\ \hline p = 18 \end{array}$$

The price of the adult ticket was \$18.

Note: Determine if the answer is reasonable.

Does a child's ticket of \$5 and an adult's ticket of \$18 cost \$23?

Yes. The answer is reasonable.

12. Step 1: Write an equation to describe the situation.

Let ℓ be the length (in cm) of the card.

$$\ell - 8 = 12$$

Step 2: Solve the equation.

$$\begin{array}{r} \ell - \cancel{8} = 12 \\ + \cancel{8} + 8 \\ \hline \ell = 20 \end{array}$$

The length is 20 cm.

Note: Determine if the answer is reasonable.

Is 20 cm minus 12 cm equal to 8 cm?

Yes. The answer is reasonable.

13. Step 1: Write an equation to describe the situation.

Let ℓ be the length (in m) of the fabric.

$$\frac{\ell}{6} = 2.75$$

Step 2: Solve the equation.

$$6\left(\frac{\ell}{6}\right) = 6(2.75) \\ \ell = 16.5$$

The original length of fabric was 16.5 m.

Note: Determine if the answer is reasonable.

If 16.5 m is divided by 6, is each piece 2.75 m?

Yes. The answer is reasonable.

14. Step 1: Write an equation to describe the equation.

Let n be the number of baseball cards Hans had originally.

$$\frac{n}{2} = 36$$

Step 2: Solve the equation.

$$2\left(\frac{n}{2}\right) = 2(36) \\ n = 72$$

Hans had 72 cards originally.

Note: Determine if the answer is reasonable.

Is one half of 72 cards equal to 36 cards?

Yes. The answer is reasonable.

15. a. Step 1: Replace the variable with the given value.

$$d = 0.12s$$

$$2.4 = 0.12s$$

Step 2: Solve the equation and answer the problem.

$$\frac{2.4}{0.12} = \frac{0.12s}{0.12} \\ 20 = s$$

The kangaroo is travelling at a speed of 20 km/h.

b. Step 1: Replace the variable with the given value.

$$d = 0.12s$$

$$4.8 = 0.12s$$

Step 2: Solve the equation and answer the problem.

$$\frac{4.8}{0.12} = \frac{0.12s}{0.12} \\ 40 = s$$

The kangaroo is travelling at a speed of 40 km/h.

16. **Step 1:** Replace the variables with the given values.

$$A = P + I$$

$$1462 = 1250 + I$$

- Step 2:** Solve the equation and answer the problem.

$$\begin{array}{r} 1462 = 1250 + I \\ -1250 \quad \cancel{-1250} \\ \hline 212 = I \end{array}$$

The interest was \$212.

17. **Step 1:** Replace the variable with the given value.

$$s = \frac{c}{2}$$

$$80 = \frac{c}{2}$$

- Step 2:** Solve the equation and answer the problem.

$$2(80) = 2\left(\frac{c}{2}\right)$$

$$160 = c$$

You should hear 160 clicks in 25 s.

Now Try This

18. **Step 1:** Make diagrams to help you solve this problem.

Note: Diagram 1 shows the train entering the tunnel. Diagram 2 shows the train after it has travelled completely through the tunnel.

Diagram 1

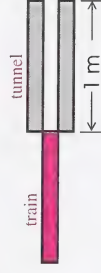


Diagram 2



The train travels 2 m to travel completely through the tunnel.

- Step 2:** Use the formula to answer the problem. Replace the variables with the given values.

$$d = st$$

$$2 = 1t$$

$$\therefore t = 2$$

The train will take 2 s to travel completely through the tunnel.

19. You can use logic to solve the problem. You could also act out the problem.

If Freddy picks two socks, they may match or they may be odd socks.

If Freddy picks three socks, there will be two of one colour and one of another colour.

Therefore, Freddy must pick three socks to be sure of getting a matching pair.

20. Use logic to solve the problem.

The boy with the black shirt replies to Billy, so Billy doesn't have a black shirt. Billy's last name is White, so Billy doesn't have a white shirt. Therefore, Billy must have a green shirt.

Ron's shirt cannot be black because his last name is Black. Therefore, Ron must have a white shirt. By elimination, Albert has the black shirt.

In summary, Billy has a green shirt, Ron has a white shirt, and Albert has a black shirt.

21. a. The formula is $c = 0.0758e$.

b. $c = 0.0758e$
 $= 0.0758(638)$
 $= 48.36$

The cost is \$48.36.

22. There are several ways you can be an energy saver.

- Compare Energiguide ratings on appliances and buy wisely.
- Set the water heater to a lower temperature.
- Keep doors, fridge and freezer doors, and windows sealed properly.
- Clean light bulbs and fixtures regularly.
- Wash full loads of laundry.
- Use the right appliance for the job (for example, do use an oven to reheat a single-portion meal).

Section 2: Activity 2

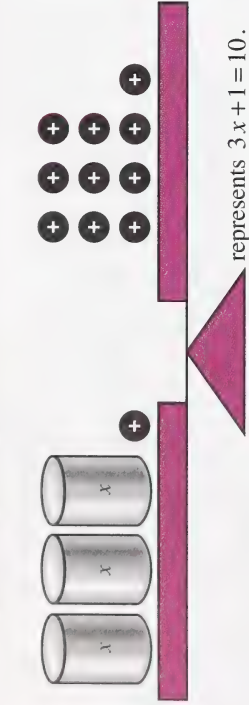
1. Check your answers by viewing the video.
2. a. **Step 1:** Isolate the variable. Add -1 to each side.

$$\frac{3x + \cancel{1} = 10}{\cancel{-1} \quad -1} \quad \frac{\quad}{3x = 9}$$

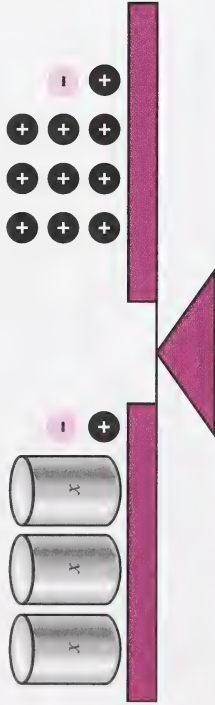
Step 2: To solve for x , divide each side by 3.

$$\frac{3x}{3} = \frac{9}{3}$$
$$x = 3$$

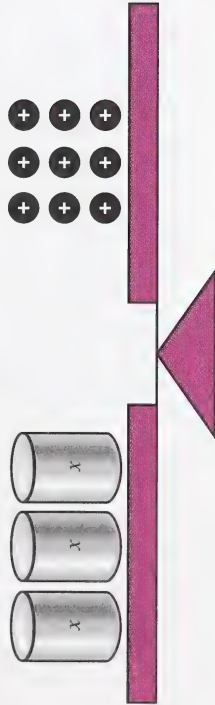
Following is an explanation of how learning aids can be used to illustrate the answer.



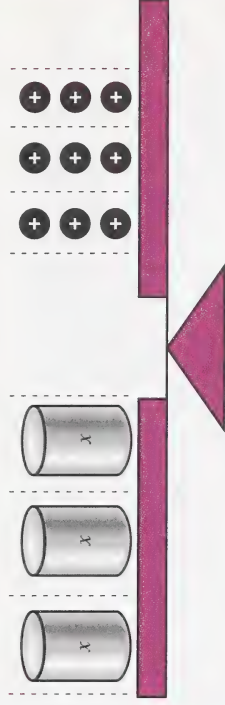
Isolate the variable. Add one negative counter to **each** side.



Simplify by removing any zero pairs. This is the result.



Arrange **each** side of the equation into three equal groups.



Examine **one** group from each side. This is the result.



The model illustrates that $x = 3$.

b. **Step 1:** Isolate the variable. Add $+3$ to each side.

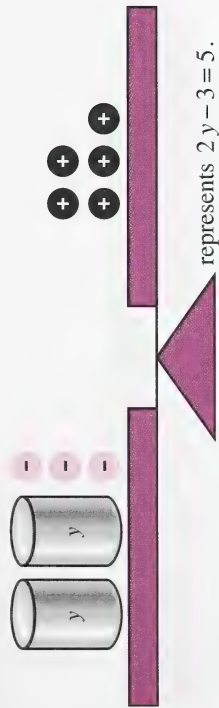
$$\begin{array}{r} 2y - 3 = 5 \\ + 3 \quad + 3 \\ \hline 2y = 8 \end{array}$$

Step 2: To solve for y , divide each side by 2.

$$\frac{2y}{2} = \frac{8}{2}$$

$$y = 4$$

Following is an explanation of how learning aids can be used to illustrate the answer.



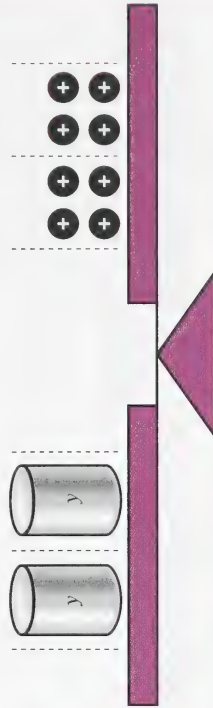
Isolate the variable. Add three positive counters to **each** side.



Simplify by removing any zero pairs. This is the result.



Arrange **each** side into two equal groups.



Examine **one** group from each side.



The model illustrates that $y = 4$.

- c. **Step 1:** Isolate the variable. Add +1 to each side.

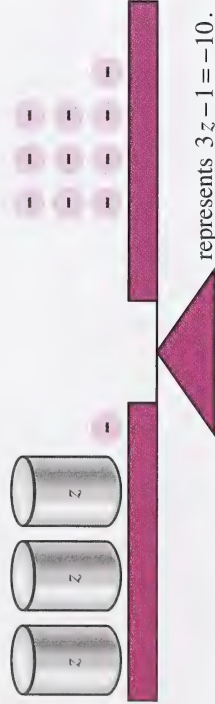
$$\begin{array}{r} 3z - 1 = -10 \\ \quad \quad \quad +1 \\ \hline 3z = -9 \end{array}$$

- Step 2:** To solve for z , divide each side by 3.

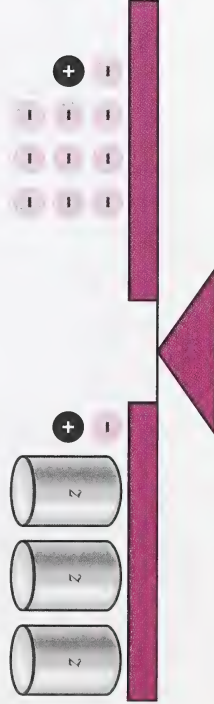
$$\frac{3z}{3} = \frac{-9}{3}$$

$$z = -3$$

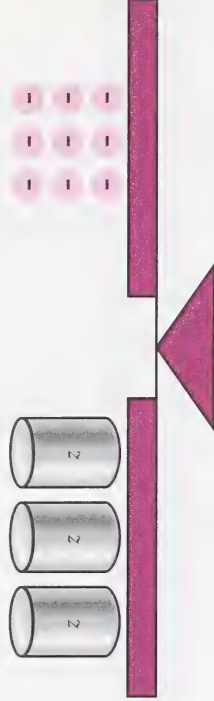
Following is an explanation of how learning aids can be used to illustrate the answer.



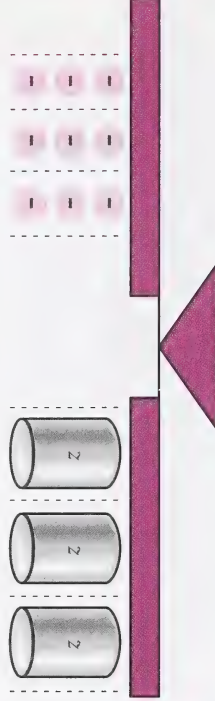
Isolate the variable. Add one positive counter to **each** side.



Simplify by removing any zero pairs. This is the result.



Arrange **each** side into three equal groups.



Examine **one** group from each side.



The model illustrates that $z = -3$.

- d. **Step 1:** Isolate the variable. Add -4 to each side.

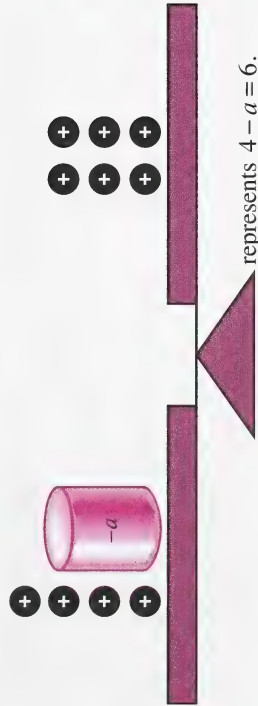
$$\begin{array}{r} -4 - a = 6 \\ -4 \quad -4 \\ \hline -a = 2 \end{array}$$

- Step 2:** To solve for a , divide each side of the equation by -1 . **Note:** a is the opposite of $-a$.

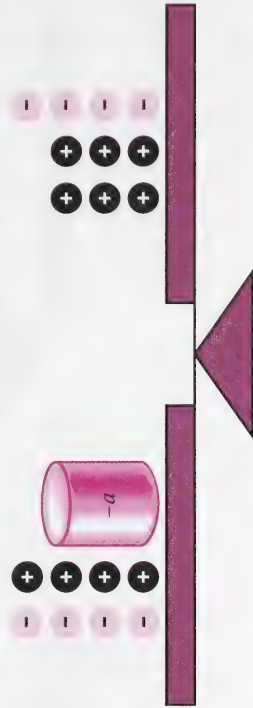
$$\begin{array}{r} -a = 2 \\ -1 \quad -1 \\ \hline a = -2 \end{array}$$

-a means $-1a$.

Following is an explanation of how learning aids can be used to illustrate the answer.



- Isolate the variable. Add four negative counters to **each** side.



Simplify by removing any zero pairs. This is the result; however, it is **not** the solution.



To solve for a , exchange the cylinders on **each** side for their opposites. **Note:** a is the opposite of $-a$; -2 is the opposite of 2 .



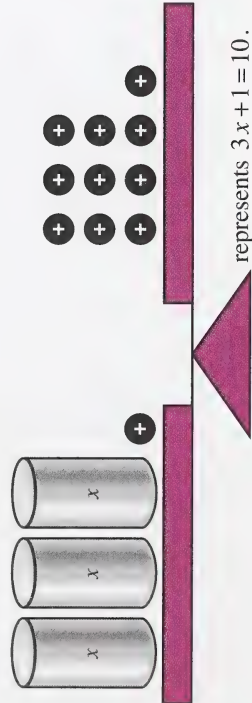
The model illustrates that $a = -2$.

3. a.

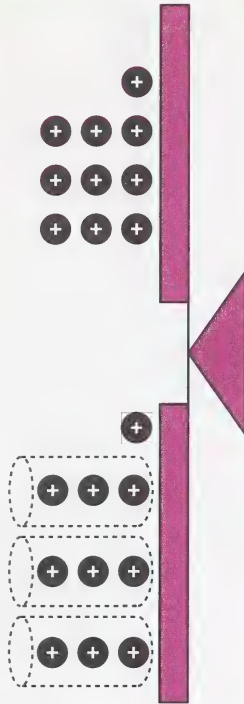
LS	RS
$3x + 1$	10
$= 3(3) + 1$	
$= 9 + 1$	
$= 10$	

LS = RS

Following is an explanation of how learning aids can be used to illustrate the answer.



To verify that $x = 3$, replace **each** cylinder with three positive counters.



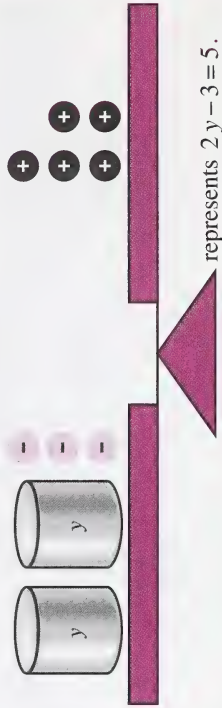
The left side equals the right side.

b.

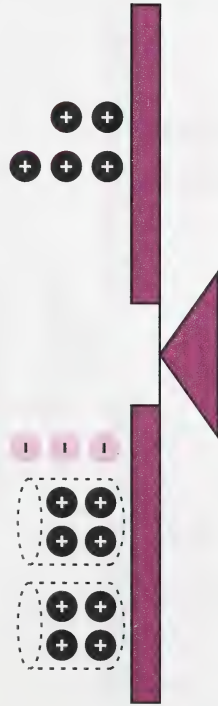
LS	RS
$2y - 3$	5
$= 2(4) - 3$	
$= 8 - 3$	
$= 5$	

LS = RS

Following is an explanation of how learning aids can be used to illustrate the answer.



To verify that $y = 4$, replace **each** cylinder with four positive counters.



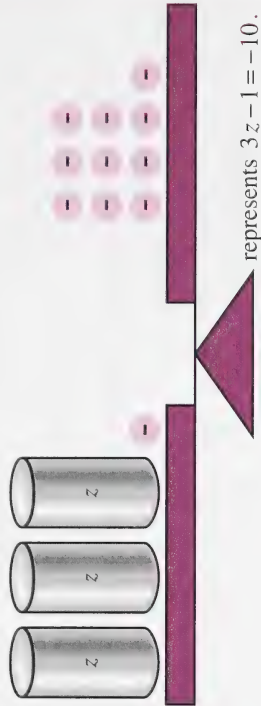
Simplify by removing any zero pairs. This is the result.



The left side equals the right side.

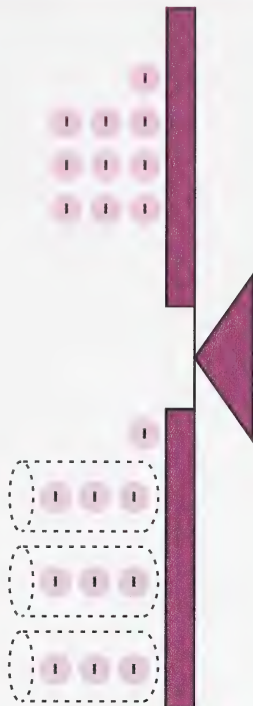
c.	LS	RS
	$3z - 1$	-10
	$= 3(-3) - 1$	
	$= -9 - 1$	
	$= -9 + (-1)$	
	$= -10$	
	LS	RS

Following is an explanation of how learning aids can be used to illustrate the answer.



represents $3z - 1 = -10$.

To verify that $z = -3$, replace **each** cylinder with three negative counters.



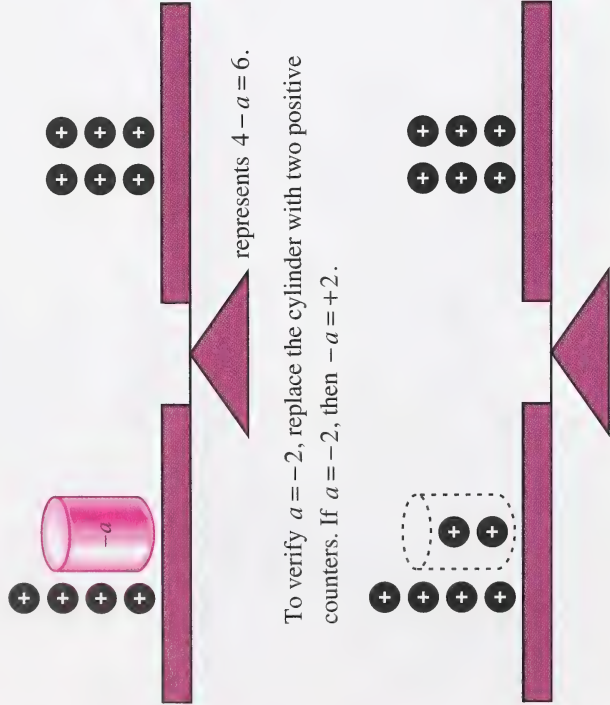
The left side equals the right side.

d.

LS	RS
$4 - a$	6
$= 4 - (-2)$	
$= 4 + 2$	
$= 6$	

LS = RS

Following is an explanation of how learning aids can be used to illustrate the answer.



The left side equals the right side.

4. a. **Step 1:** Write an equation to describe the situation.

Let a be Avi's age (in a).

$$29 - a = 12$$

Step 2: Isolate the variable.

$$\begin{array}{r} \cancel{29} - a = 12 \\ -\cancel{29} \quad -29 \\ \hline -a = -17 \end{array}$$

$-a$ means $-1a$.

Step 3: Solve for a and answer the problem.

Note: a is the opposite of $-a$.

$$\begin{array}{r} -a \quad -17 \\ -1 \quad -1 \\ \hline a = 17 \end{array}$$

Avi is 17 years old.

Note: Check the answer.

Effi is 29 years old and older than Avi. If Avi is 17 years old, is the difference in their ages 12 years?
Yes, $29 - 17 = 12$.

The answer is reasonable.

b. Step 1: Write an equation to describe the situation.

Let n represent the number of rides.

$$5 + 2n = 25$$

Step 2: Isolate the variable in the equation.

$$\begin{array}{r} 5 + 2n = 25 \\ -5 \\ \hline 2n = 20 \end{array}$$

Step 3: Solve for n and answer the problem.

$$\begin{array}{r} 2n = 20 \\ \div 2 \\ \hline n = 10 \end{array}$$

Mavis went on 10 rides.

Note: Check the answer.

Does \$5 plus 10 times \$2 equal \$25? Yes, $5 + 10 \times 2 = 25$.

The answer is reasonable.

c. Step 1: Write an equation to describe the situation.

Let n be the number of cookies per bag.

$$3n - 39 = 15$$

Step 2: Isolate the variable in the equation.

$$\begin{array}{r} 3n - 39 = 15 \\ +39 \\ \hline 3n = 54 \end{array}$$

Step 3: Solve for n and answer the problem.

$$\begin{array}{r} 3n = 54 \\ \div 3 \\ \hline n = 18 \end{array}$$

Each bag had 18 cookies.

Note: Check the answer.

If each bag had 18 cookies, then 3 bags had 54 cookies. If the teens ate 39 cookies, would there be 15 cookies left? Yes, $54 - 39 = 15$.

The answer is reasonable.

d. **Step 1:** Write an equation to describe the situation.

Let h be yesterday's high temperature (in $^{\circ}\text{C}$).

$$3h + 5 = -31$$

Step 2: Isolate the variable in the equation.

$$\begin{array}{r} 3h + \cancel{5} = -31 \\ \cancel{-5} \quad -5 \\ \hline 3h = -36 \end{array}$$

Step 3: Solve for h and answer the problem.

$$\begin{array}{r} 3h = -36 \\ \hline 3 \quad 3 \\ h = -12 \end{array}$$

Yesterday's high temperature was -12°C .

Note: Check the answer.

Is five more than three times -12°C equal to -31°C ?

Yes, $3(-12) + 5 = -31$.

The answer is reasonable.

e. **Step 1:** Write an equation to describe the situation.

Let n be the amount (in $\$$) of each deposit.

$$24 + 4n = 72$$

Step 2: Isolate the variable in the equation.

$$\begin{array}{r} \cancel{24} + 4n = 72 \\ \cancel{-24} \quad -24 \\ \hline 4n = 48 \end{array}$$

Step 3: Solve for n and answer the problem.

$$\begin{array}{r} 4n = 48 \\ \hline 4 \quad 4 \\ n = 12 \end{array}$$

Each deposit was $\$12$.

Note: Check the answer.

If Troy made 4 equal deposits of $\$12$, he deposited $\$48$ altogether. Does $\$24$ plus $\$48$ equal $\$72$. Yes, $24 + 48 = 72$.

The answer is reasonable.

f. **Step 1:** Write an equation to describe the situation.

Let n be the number of hours.

$$3 + 2n = 15$$

Step 2: Isolate the variable in the equation.

$$\begin{array}{r} 3 + 2n = 15 \\ -3 \quad -3 \\ \hline 2n = 12 \end{array}$$

Step 3: Solve for n and answer the problem.

$$\begin{array}{r} 2n = 12 \\ 2 \quad 2 \\ \hline n = 6 \end{array}$$

Maria can rent skates for 6 h.

Note: Check the answer.

It costs \$15 to rent skates for 6 h. Maria has \$15. Is 6 h the maximum time for which Maria can rent skates? Yes, she will have spent all her money.

The answer is reasonable.

5. **Step 1:** Substitute 600 for n in the formula.

$$\begin{array}{l} n = 60a - 900 \\ 600 = 60a - 900 \end{array}$$

Step 2: Isolate the variable.

$$\begin{array}{r} 600 = 60a - 900 \\ +900 \quad +900 \\ \hline 1500 = 60a \end{array}$$

Step 3: Solve for a and answer the question.

$$\begin{array}{r} 1500 = 60a \\ 60 \quad 60 \\ \hline 25 = a \end{array}$$

On average a child who uses 600 words correctly is about 25 months old.

6. a. **Step 1:** Substitute 272 for h in the formula.

$$\begin{array}{l} h = 1.88f + 81.3 \\ 272 = 1.88f + 81.3 \end{array}$$

Step 2: Isolate the variable.

$$\begin{array}{r} 272 = 1.88f + 81.3 \\ -81.3 \quad -81.3 \\ \hline 190.7 = 1.88f \end{array}$$

Step 3: Solve for f and answer the question.

$$\frac{190.7}{1.88} = \frac{1.88f}{1.88}$$

$$101 \div f$$

Robert Wadlow's femur would have been about 101 cm long. (That is, the femur was about 1.01 m long.)

b. Step 1: Substitute 248 for h in the formula.

$$h = 1.95f + 73$$

$$248 = 1.95f + 73$$

Step 2: Isolate the variable.

$$\begin{array}{r} 248 = 1.95f + \cancel{73} \\ -73 \quad \quad \quad \cancel{-73} \\ \hline 175 = 1.95f \end{array}$$

Step 3: Solve for f and answer the question.

$$\frac{175}{1.95} = \frac{1.95f}{1.95}$$

$$90 \div f$$

Zeng Jinlian's femur would have been about 90 cm long.

7. a. Step 1: Isolate the variable.

$$\frac{n+1}{6} = 2$$

$$\frac{\cancel{n}+1}{6} = 2$$

$$\frac{n}{6} = 1$$

Step 2: Solve for n .

$$6\left(\frac{n}{6}\right) = 6(1)$$

$$n = 6$$

b. Step 1: Isolate the variable.

$$\frac{d-\cancel{5}}{3} = 1$$

$$\frac{\cancel{d}-5}{3} = 1$$

$$\frac{d}{3} = 6$$

Step 2: Solve for d .

$$3\left(\frac{d}{3}\right) = 3(6)$$

$$d = 18$$

LS	RS
$\frac{n+1}{6}$	2
$= \frac{6+1}{6}$	
$= 1+1$	
$= 2$	

$$LS = RS$$

LS	RS
$\frac{d-5}{3}$	1
$= \frac{18-5}{3}$	
$= \frac{6-5}{3}$	
$= 1$	

$$LS = RS$$

c. **Step 1:** Isolate the variable.

$$\begin{array}{r} 3 - \frac{m}{2} = 9 \\ \underline{-3} \\ -\frac{m}{2} = 6 \\ \underline{\times 2} \\ -m = 12 \\ \underline{\times -1} \\ m = -12 \end{array}$$

Step 2: Solve for m .

$$\begin{array}{l} 2\left(\frac{-m}{2}\right) = 2(6) \\ -m = 12 \\ m = -12 \end{array}$$

d. **Step 1:** Isolate the variable.

$$\begin{array}{r} 5 - \frac{r}{2} = -11 \\ \underline{-5} \\ -\frac{r}{2} = -16 \\ \underline{\times 2} \\ -r = -32 \\ \underline{\times -1} \\ r = 32 \end{array}$$

Step 2: Solve for r .

$$\begin{array}{l} 2\left(\frac{-r}{2}\right) = 2(-16) \\ -r = -32 \\ r = 32 \end{array}$$

8. a. **Step 1:** Write an equation to describe the situation.

Let n be the number of paper clips Vancho had originally.

$$\frac{n}{2} - 7 = 23$$

Step 2: Isolate the variable.

$$\begin{array}{r} \frac{n}{2} - 7 = 23 \\ \underline{+7} \\ \frac{n}{2} = 30 \\ \underline{\times 2} \\ n = 60 \end{array}$$

Step 3: Solve for n and answer the question.

$$\begin{array}{l} 2\left(\frac{n}{2}\right) = 2(30) \\ n = 60 \end{array}$$

Vancho had 60 paper clips originally.

Note: Check the answer.

If Vancho had 60 paper clips and gave Vera half of his paper clips, he would have given her 30 clips. If she lost 7 paper clips, would she have 23 left? Yes, $30 - 7 = 23$.

The answer is reasonable.

b. **Step 1:** Write an equation to describe the situation.

Let n be the age (in years) of Shannon's father.

$$\frac{n}{3} + 2 = 14$$

Step 2: Isolate the variable.

$$\begin{array}{r} \frac{n}{3} + \cancel{2} = 14 \\ \cancel{-2} \quad -2 \\ \hline \frac{n}{3} = 12 \end{array}$$

Step 3: Solve for n and answer the question.

$$\begin{array}{l} 3\left(\frac{n}{3}\right) = 3(12) \\ n = 36 \end{array}$$

Shannon's father is 36 years old.

Note: Check the answer.

Suppose Shannon is 14 years old and Shannon's father is 36 years old. Is Shannon's present age 2 years more than one-third of her father's age? Yes, $\frac{1}{3} \times 36 + 2 = 14$.

The answer is reasonable.

c. **Step 1:** Write an equation to describe the situation.

Let m be the average mass (in kg) of a coyote.

$$\frac{m}{2} + 1 = 9$$

Step 2: Isolate the variable.

$$\begin{array}{r} \frac{m}{2} + \cancel{1} = 9 \\ \cancel{-1} \quad -1 \\ \hline \frac{m}{2} = 8 \end{array}$$

Step 3: Solve for m and answer the question.

$$\begin{array}{l} 2\left(\frac{m}{2}\right) = 2(8) \\ m = 16 \end{array}$$

The average mass of a coyote is 16 kg.

Note: Check the answer.

Suppose the average mass of a coyote is 16 kg and the average mass of a raccoon is 9 kg. Is the mass of the raccoon 1 kg more than one-half the mass of the coyote?

Yes. The answer is reasonable.

9. **Step 1:** Substitute 25 for t in the given formula.

$$t = \frac{n}{8} + 5$$

$$25 = \frac{n}{8} + 5$$

- Step 2:** Isolate the variable.

$$25 = \frac{n}{8} + 5$$

$$-5 = \frac{n}{8}$$

$$20 = \frac{n}{8}$$

- Step 3:** Solve for n and answer the question.

$$8(20) = 8\left(\frac{n}{8}\right)$$

$$160 = n$$

A cricket should chirp 160 times in a minute.

10. **Step 1:** Substitute 10 for t in the given formula.

$$t = 15 - \frac{d}{4}$$

$$10 = 15 - \frac{d}{4}$$

- Step 2:** Isolate the variable.

$$10 = 15 - \frac{d}{4}$$

$$-15 = -\frac{d}{4}$$

$$-5 = \frac{-d}{4}$$

- Step 3:** Solve for n and answer the question.

$$4(-5) = 4\left(\frac{-d}{4}\right)$$

$$-20 = -d$$

$$20 = d$$

The temperature is 10°C at 20 m.

11. **Step 1:** Write an equation to describe the situation.

Let r be the regular price (in \$).

$$\frac{3r}{4} = 6$$

- Step 2:** Clear the denominator by multiplying each side by 4.

$$\frac{3r}{4} = 6$$

$$4\left(\frac{3r}{4}\right) = 4(6)$$

$$3r = 24$$

Step 3: Solve for r and answer the question.

$$\frac{3r}{3} = \frac{24}{3}$$
$$r = 8$$

The regular price is \$8.

Note: Check the answer.

$$\text{Is } \frac{3}{4} \times 8 = 6?$$

Yes, the answer is reasonable.

12. Step 1: Write an equation to describe the situation.

Let r be the regular price (in \$).

$$\frac{2r}{3} = 12$$

Step 2: Clear the denominator by multiplying each side by 3.

$$3\left(\frac{2r}{3}\right) = 3(12)$$
$$2r = 36$$

Step 3: Solve for r and answer the question.

$$\frac{2r}{2} = \frac{36}{2}$$
$$r = 18$$

The regular price is \$18.

Note: Check the answer.

Suppose the regular price of the sweater is \$18. If the sweater was on sale for one-third off, would the sale price be \$12?

$$\text{Yes, } \frac{2}{3} \times 18 = 12.$$

The answer is reasonable.

Now Try This

- 13. a.** Answers will vary. Following are three examples of problems that can be made from the given information.

Problem 1: It is 300 km from Regina to Gull Lake. About halfway between the two locations is Chaplin. How far is it from Regina to Chaplin?

Problem 2: It is 300 km from Regina to Gull Lake. Delaho drives her car the speed limit of 100 km/h. How long does it take Delaho to drive from Regina to Gull Lake?

Problem 3: Delaho drives her car the speed limit of 100 km/h. Alain drives his car 10 km/h slower than Delaho. At what speed is Alain travelling?

- b. Answers will vary. Following are the solutions to the problems in question 13.a.

Problem 1

Let d be the distance (in km) from Regina to Chaplin.

$$d = \frac{1}{2}(300)$$

$$d = 150$$

It is 150 km from Regina to Chaplin.

Problem 2

Use the distance formula, $d = st$.

$$d = st$$

$$300 = 100t$$

$$\frac{300}{100} = \frac{100t}{100}$$

$$3 = t$$

It takes Delaho 3 h to drive from Regina to Gull Lake.

Problem 3

Let s represent the speed (in km/h) at which Alain is travelling.

$$100 - 10 = s$$

$$90 = s$$

Alain is travelling at a speed of 90 km/h.

Did You Know?

14. Use the guess, check, and revise strategy and logic to solve these cryptarithms. Here are two possible solutions. Other solutions are possible.

$$\begin{array}{r} \text{SEND} \\ + \text{MORE} \\ \hline \text{MONEY} \end{array} = \begin{array}{r} 9\ 567 \\ + \ 1\ 085 \\ \hline 10\ 652 \end{array}$$

$$\begin{array}{r} \text{SPEND} \\ - \text{LESS} \\ \hline \text{DEAR} \end{array} = \begin{array}{r} 13\ 075 \\ - \ 8\ 011 \\ \hline 5\ 064 \end{array}$$

Section 2: Follow-up Activities

Extra Help

1.

Doing Action	Undoing Action
Open a door.	Close the door.
Turn on a tap.	Turn off the tap.
Put on socks and then put on shoes.	Take off shoes and then take off socks.
Unload cargo from a truck and put cargo on a boat.	Take cargo off the boat and load the cargo on a truck.

2. a. Flow Chart

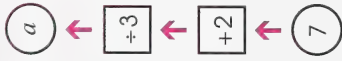


$$\therefore a = 3$$

Note: Verify the solution.

LS	RS
$3a - 2$ $= 3(3) - 2$ $= 9 - 2$ $= 7$	7
LS	= RS

Reverse Flow Chart



b. Flow Chart



$$\therefore b = -5$$

Note: Verify the solution.

LS	RS
$2b + 1$ $= 2(-5) + 1$ $= -10 + 1$ $= -9$	-9
LS	= RS

Reverse Flow Chart



c. Flow Chart



$$\therefore c = 27$$

Note: Verify the solution.

LS	RS
$\frac{c-4}{3}$ $= \frac{(27)-4}{3}$ $= 9-4$ $= 5$	5
LS	= RS

Reverse Flow Chart



d. Flow Chart

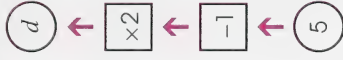


$$\therefore d = 8$$

Note: Verify the solution.

LS	RS
$\frac{d+1}{2}$ $= \frac{(8)+1}{2}$ $= 4+1$ $= 5$	5
LS	= RS

Reverse Flow Chart



Enrichment

1. Step 1: Make a chart and establish a pattern for the cost. **Note:**

There is a flat charge of \$2.48 for the first minute.

Time Talked (min)	Cost (\$)
1	2.48
2	4.13
3	5.78
4	7.43
...	...
t	$2.48 + 1.65(t - 1)$

$$\begin{aligned} \rightarrow c &= 2.48 + 1.65(2 - 1) \\ \rightarrow c &= 2.48 + 1.65(3 - 1) \\ \rightarrow c &= 2.48 + 1.65(4 - 1) \end{aligned}$$

Step 2: Write a formula to explain the situation. Let c be the cost (in \$) and t be the time of the call (in min).

$$c = 2.48 + 1.65(t - 1)$$

Step 3: Start a new spreadsheet document and enter the following data and formulas to calculate the cost of a sequence of calls.

A		B	
1	Time Talked (min)	Cost (\$)	
2		1	2.48
3	=A2+1		=2.48+1.65*(A3-1)
4	=A3+1		=2.48+1.65*(A4-1)
...			

Step 4: Fill the cells in columns A and B by doing the following:

- Highlight cell A3 and then drag down the column, or click on cell A3 and shift-click on the last cell of the column. Next, click on "Calculate" in the menu bar and select "Fill Down."
- Highlight cell B3 and then drag down the column, or click on cell B3 and shift-click on the last cell of the column. Next, click on "Calculate" in the menu bar and select "Fill Down."

Step 5: To solve the problem, find the highest number equal to or less than 20 in column B and read the corresponding number in column A.

A		B	
1	Time talked (min)	Cost (\$)	
2		1	2.48
3		2	4.13
4		3	5.78
5		4	7.43
6		5	9.08
7		6	10.73
8		7	12.38
9		8	14.03
10		9	15.68
11		10	17.33
12		11	18.98
13		12	20.63

Examine rows 12 and 13.

Jonathan can talk for 12 min.

2. Step 1: Make a chart and establish a pattern for the cost. **Note:**

There is a flat charge of \$7.95 for a pizza with two toppings.

Number of Toppings (n)	Cost (\$)
2	7.95
3	8.80
4	9.65
5	10.50
...	...
n	$7.95 + 0.85(n - 2)$

$$\leftarrow c = 7.95 + 0.85(3 - 2)$$

$$\leftarrow c = 7.95 + 0.85(4 - 2)$$

$$\leftarrow c = 7.95 + 0.85(5 - 2)$$

Step 2: Write a formula to explain the situation. Let c be the cost (in \$) and n be the number of pizza toppings.

$$c = 7.95 + 0.85(n - 2)$$

Step 3: Start a new spreadsheet document and enter the following data and formulas to calculate the cost of a sequence of pizzas.

	A	B
1	Number of Toppings	Cost
2	2	7.95
3	$=A2+1$	$=7.95+0.85*(A3-1)$

Step 4: Fill the cells in columns A and B by doing the following:

- Highlight cell A3 and then drag down the column, or click on cell A3 and shift-click on cell A7. Next, click on "Calculate" in the menu bar and select "Fill Down."
- Highlight cell B3 and then drag down the column, or click on cell B3 and shift-click on cell B7. Next, click on "Calculate" in the menu bar and select "Fill Down."

Step 5: To solve the problem, find the highest number equal to or less than 12 in column B and read the corresponding number in column A.

	A	B
1	Number of Toppings	Cost
2	2	7.95
3	3	9.65
4	4	10.5
5	5	11.35
6	6	12.2
7	7	13.05

Examine rows 5 and 6.

Jocelyn and her friends can order maximum a total of 5 toppings.

The Wheat That Created the Canadian West

Red Fife

Date: 1843

Discoverer: David Fife

The year is 1843. The Union act has just recently been ratified. In Peterborough, a young farmer of Scottish origin, David Fife, has been devoting himself to experiments which have not gone unnoticed by his somewhat amused neighbours.

Seeking a strain of wheat more suited to the Canadian climate, Fife planted some seeds he had received through the mail from a Scottish correspondent. Much to his disappointment, he confirmed that this variety was an "autumn" wheat which grew slowly, and therefore was destroyed by the first frost. In fact, Fife's entire crop was lost, with the exception of one plant. This super-resistant plant (for that time) attained maturity and gave birth to the celebrated Red Fife.

Baptized thus with the name of its creator, this wheat became famous for its ability to ripen days earlier than any other known variety. It was the first Canadian wheat of world quality; in addition to its precocity, it was high-yielding, rich and excellent for making bread. David Fife's perseverance had made him famous.

Marquis Wheat

Date: 1908

Discoverer: Charles Saunders

In Ottawa a man with astonishing energy, William Saunders, became the first director of the Dominion Experimental Farms in 1884. For several years, Saunders had been crossing Red Fife with every imaginable variety of wheat, with little success. Gradually, his position as director imposed responsibilities on him which allowed little time for his experiments. What was he to do? He did not want to abandon them; they were too important. Dominating father that he was, he created a position for his musician son, Charles, who became the first Dominion Cerealists.

Young Saunders arrived at a laboratory lined with hundreds of glass bottles of earth and seeds—his father's legacy. Anyone else would have thrown it all out and started over. Not Charles. Lacking enthusiasm, but endowed with a keen organizational sense, he patiently took up the work of creating wheat hybrids. And one day, having crossed Red Fife with an Indian variety called Hard Red Calcutta, Saunders created Marquis wheat.

Marquis was more resistant to disease than Red Fife, produced more grains to the bushel and, best of all, was ready for harvest in 100 days. The first year it was seriously tested, 1907, was especially short. Frosts came early and the harvest was a disaster for everyone except farmers who had planted Marquis. Commercially launched in 1908, this strain captured over 90 percent of the wheat lands within 12 years.

The First Transmission of Electrical Power at 735 kV

Date: 1965

Innovator: Hydro-Québec, led by two of its engineers, Jean-Jacques Archambault and Lionel Cahill

Significance: Raising voltages lowers the cost of transmitting power.

Profile: Glaciers long ago sculpted the surface of Quebec into vast, shallow, rocky basins. An enormous quantity of water is caught in these basins, from which it flows through streams and rivers down to the sea. This phenomenon of nature was converted into a phenomenal source of renewable energy by the development of technology for transforming the energy of running water into electricity.

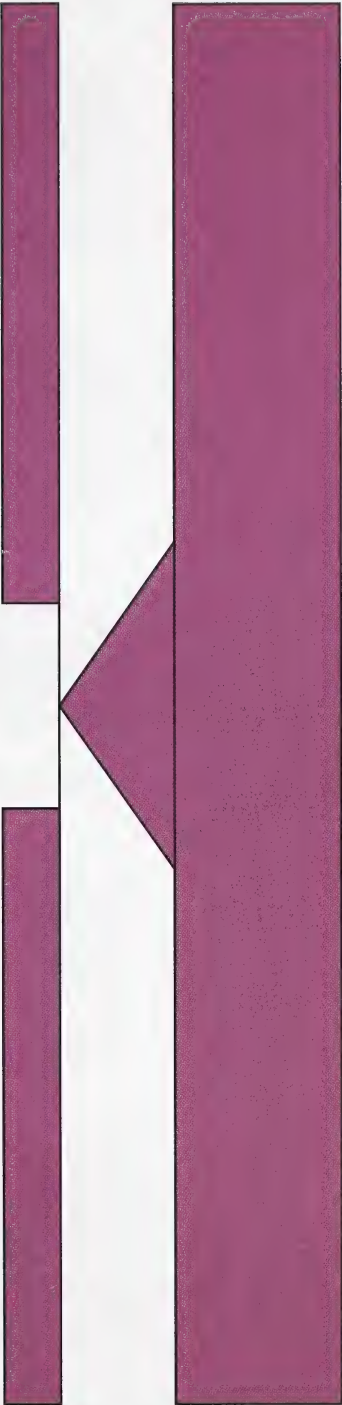
Since the beginning of this century, the amount of electricity generated from Quebec's rivers has doubled every ten years or so, an explosive rate of growth. The first dams and power plants, naturally, were built at the biggest, or most convenient sites: on rivers near cities where power is used, or on rivers from which such abundant power could be generated that it was economical to build electricity-hungry industrial plants, such as aluminum smelters, on their banks. By the 1960s however, Hydro-Québec, a provincial government corporation, was reaching out to control wild, far-distant rivers.

At that time the standard voltage for long distance power transmission was 315 kV, but it was clear that higher voltages would be needed to move power from the powerhouses of the Manic-Outardes complex, then under construction some 640 km northeast of Montreal. Conventional wisdom had it that Hydro-Québec should raise the voltage for the Manic lines to 525 kV. But Jean-Jacques Archambault was sure that 735 kV lines were feasible, and that pioneering them now would make sense later, when Hydro-Québec might want to plug power from even more distant sites, such as Churchill Falls and James Bay, into its grid. He convinced his colleague, Lionel Cahill, of the merits of 735 kV. After three furious years of tests, plans, contracting and constructing, the first 735 kV line in the world was electrified on September 21, 1965. It failed: a painter, it turned out, had inadvertently connected a conductor to a pylon. This short-circuit was quickly traced and, to the relief of the anxious engineers, the line was successfully brought up to full voltage.

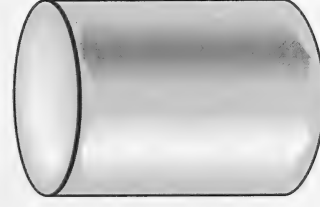
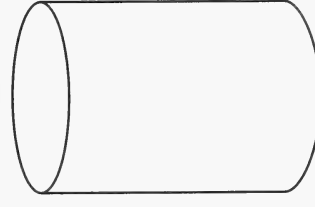
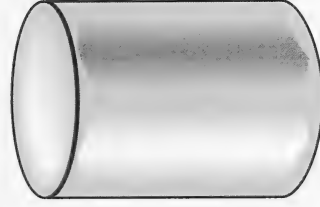
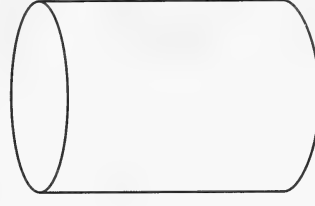
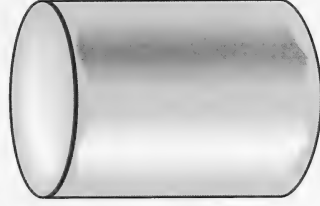
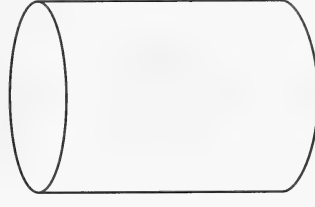
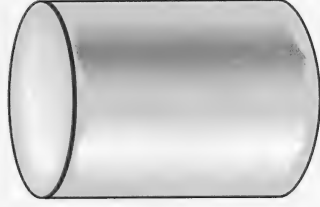
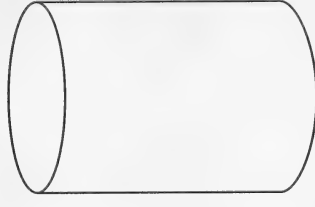
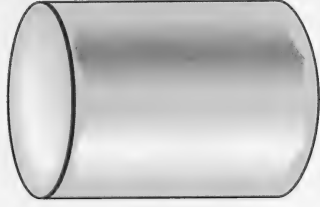
Why High Voltages for Long Distance Power Transmission?

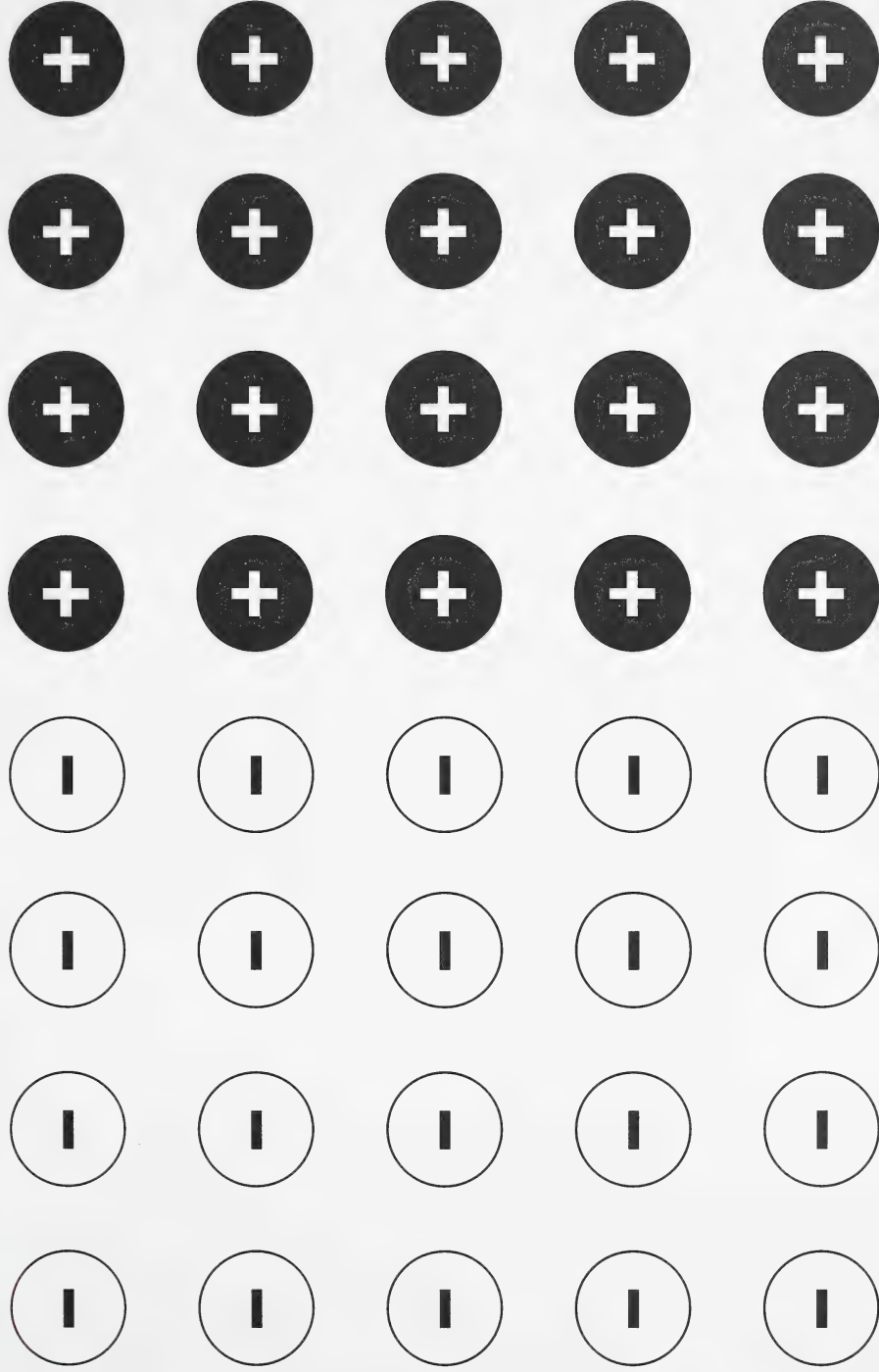
Voltage, the force that "pushes" electricity through a wire, is analogous to pressure, the force that pushes water through a pipe. The greater the pressure, the more water is forced through a given pipe. So too, the higher the voltage, the more electrical energy is transmitted. For a given distance, a 735-kV line costs about twice as much as a 315 kV one, but carries four times more power.

Equation Scale



Cylinders





Student and teacher: Use this cover sheet for mailing or faxing.

ASSIGNMENT BOOKLET

8110 Mathematics 8
Module 2

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**Student's Questions
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Teacher

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- Has your work been reread to ensure accuracy in spelling and details?
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MATHEMATICS 8

MODULE 2



Patterns and Relations

ASSIGNMENT BOOKLET

FOR TEACHER'S USE ONLY

Summary

	Total Possible Marks	Your Mark
Section 1 Assignment	40	
Section 2 Assignment	50	
Final Module Assignment	10	
	100	

Teacher's Comments

This document is intended for

Students	✓
Teachers	✓
Administrators	
Parents	
General Public	
Other	

Mathematics 8
 Assignment Booklet
 Module 2
 Patterns and Relations
 Learning Technologies Branch
 ISBN 0-7741-1334-0

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ASSIGNMENT BOOKLET

MATHEMATICS 8 – MODULE 2: PATTERNS AND RELATIONS

Your mark on this module will be determined by how well you do your assignments in this booklet.

Work slowly and carefully. If you are having difficulties, go back and review the appropriate section.

There are two section assignments and one final module assignment in this Assignment Booklet. The total value of these assignments is 100 marks. The value of each assignment is stated in the left margin.

You may do your rough work on your own paper.

Be sure to proofread each assignment carefully.

40

Section 1 Assignment: Language of Algebra

Read all the parts of your assignment carefully and record your answers in the appropriate place.

1. Write an algebraic expression to describe the total number of jellybeans in each of the following pictures. **Note:** Each jar has the same number of jellybeans. You do not know the number of jellybeans in each jar.

a.



b.



2. Write an algebraic expression for the following phrase:

three less than one-half of a number _____

3. Write a phrase for each of the following algebraic expressions.

2

a. $3n + 1$ _____

2

b. $5 - 9x$ _____

4. When a new drug is developed and the recommended dosage for adults is determined, a corresponding dosage for children may be calculated.

Friend’s rule is sometimes used to calculate the different dosages for children under two years of age.

Following is the formula for Friend’s rule. In this formula, c is the recommended dosage for children **under the age of two years**, m is the child’s age in months, and a is the adult dosage.

$$c = \frac{am}{150}$$

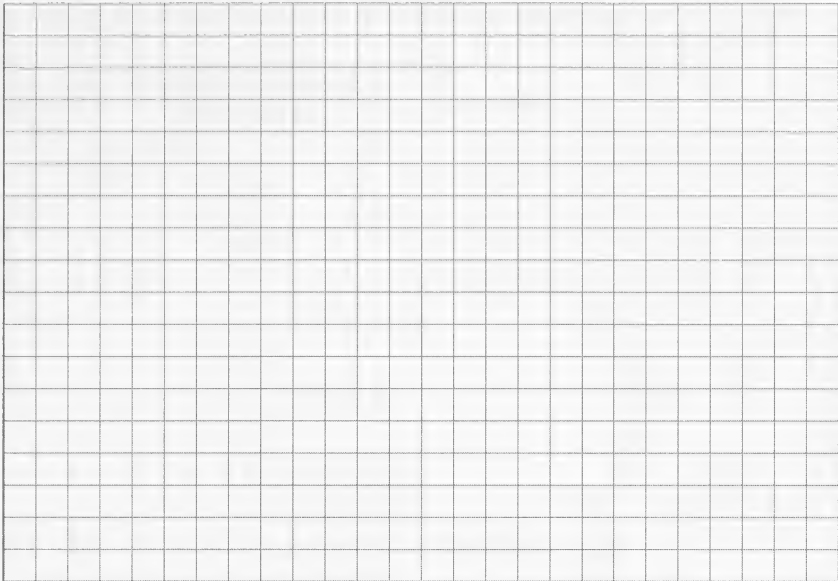
4

a. Suppose the adult dosage of a particular medicine is 50 mg. Use the formula for Friend’s rule to complete the following chart. **Note:** Space is provided to show your work.

Age of Child (in months)	Recommended Child’s Dosage (in mg)
3	
6	
9	
12	
15	
18	

4

b. Make a graph showing how the recommended child’s dosage is related to the child’s age.



2

c. How is the child’s dosage affected by the child’s age?

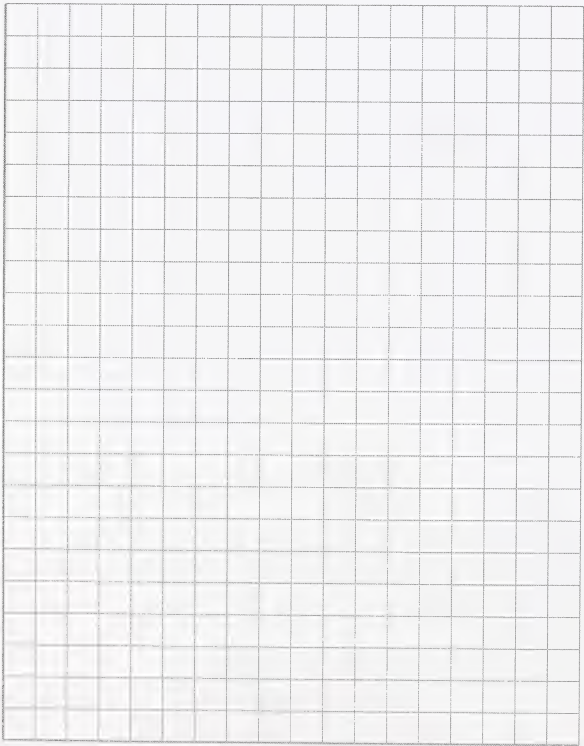
5. The daylight hours are those between sunrise and sunset. For one year, Francine kept track of the number of daylight hours on the 21st day of each month. She recorded the number of daylight hours (rounded to the nearest half hour) in the following chart.

Month	Number of Hours of Daylight
1	9.5
2	10.5
3	12
4	14
5	15
6	15.5
7	15
8	14
9	12
10	10.5
11	9.5
12	9

Note: January is the 1st month, February is the 2nd month, March is the 3rd month, and so on.

4

- a. Make a graph to show how the number of daylight hours is related to the time of the year.



①

- b. What month had the most daylight hours?

①

- c. What month had the least daylight hours?

②

- d. How is the number of daylight hours affected by the time of the year?

②

6. a. Give the next two terms in this sequence:

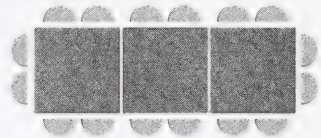
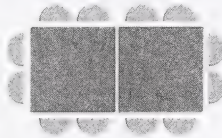
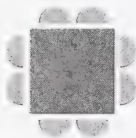
7, 9, 11, 13, ...

②

- b. Write a formula to describe the n th term in this sequence. **Note:** Space is provided to show your work.

- 2 c. Use the formula to predict the 20th term of the sequence. **Note:** Space is provided to show your work.

7. Marco works in a restaurant. When longer tables are needed, square tables are placed together. The following pictures show the pattern that is used.



- 2 a. Draw a picture to show how many people can be seated if 4 tables are placed together in this pattern.

- 2 b. Write a formula to describe how the number of seats is related to the number of tables. **Note:** Space is provided to show your work.

2

- c. Use the formula to predict how many people can be seated if 10 tables are placed together. **Note:** Space is provided to show your work.

50

Section 2 Assignment: Equations

Read all the parts of your assignment carefully and record your answers in the appropriate place. When answering the following questions, be sure to clearly show how you arrived at your answer.

1. Solve each of the following equations algebraically. Then verify each of the solutions.

3

a. $x + 2 = 5$

3

b. $3x = 18$

③

c. $\frac{m}{4} = 2$

⑤

d. $3n - 1 = 8$

⑤

e. $\frac{r}{2} + 3 = 7$

⑤

f. $\frac{3t}{4} = -6$

2. Martin is having difficulty solving the equation $2x - 3 = 5$ algebraically.

7

- a. To help Martin understand the steps in solving the equation $2x - 3 = 5$ algebraically, use models. Write an explanation and draw diagrams of models.

③

- b. Solve the equation $2x - 3 = 5$ algebraically.

3. Use equations to solve each of the following problems.

⑤

- a. A photo lab charges \$12 for developing a roll of 24 exposures. The lab charges \$2 for each enlargement. Sergei had a roll of film developed and some enlargements made. His total bill was \$20. How many enlargements were made? **Note:** You do not have to consider GST.

⑤

- b. A portable radio is on sale for $\frac{1}{3}$ of its regular price. Yolanda purchased the radio and also used a coupon for \$5 off. If Yolanda's final bill was \$50, what was the regular price of the radio? **Note:** You do not have to consider GST.

3

4. a. Following is some information about the Chang family.

There are four children in the Chang family. Michael is twice Peter's age. Carla is 5 years older than Michael. Gina is 3 years younger than Michael. Michael is 14.

Use the information to write a problem. **Note:** You do **not** need to use **all** the information.

3

- b. Use an equation to solve the problem that you wrote in question 4.a.

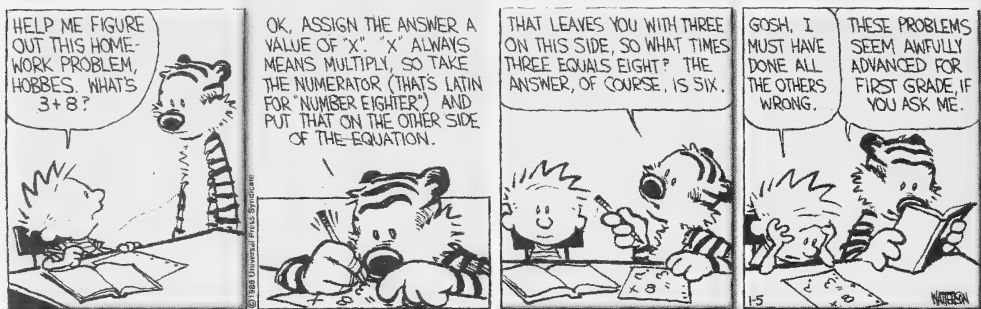
10

Final Module Assignment

Read all the parts of your assignment carefully and record your answers in the appropriate place.

Use the following “Calvin and Hobbes” cartoon to answer question 1.

Calvin and Hobbes by Bill Watterson



2

1. a. Hobbes makes many errors in this cartoon. For example, he states that “ $\times$ ” always means multiply. What else does “ $\times$ ” mean?

2

- b. Hobbes asks, “So what times three equals eight?” He then says, “The answer, of course, is six.” Translate Hobbes’s question into an equation and solve the equation. Did Hobbes answer his own question correctly?

②

2. a. The phrase “five more than a number” can be written as “ $n + 5$ ” and also as “ $5 + n$.”

Can the phrase “five less than a number” be written as “ $n - 5$ ” and also as “ $5 - n$ ”?
Explain using drawings of models.

②

- b. Confirm your answer to question 2.a. by evaluating $n - 5$ and $5 - n$, if $n = 1$.

②

3. Tina translated the expression $3x - 2$ as “three times a number minus two.”

Tell Tina why her sentence might be confusing. Give another way her sentence might be interpreted. Explain how to make her words clearer.

ASSIGNMENT BOOKLET DECLARATIONS

The Student's Declaration is to be filled in by a student registered at the Alberta Distance Learning Centre. If the student is under 16, the Learning Facilitator's Declaration is to be filled in by the learning facilitator. Failure to complete this page may invalidate the assignment results.

STUDENT'S DECLARATION

- I have followed the instructions outlined in the Student Module Booklet.
- I have completed the activities to prepare myself for the assignments in this Assignment Booklet.
- I completed the assignments in this Assignment Booklet by myself.

Student's Signature

LEARNING FACILITATOR'S DECLARATION

I hereby certify that I have supervised the learning activities completed by _____.
Student's Name

I also certify that to the best of my knowledge the assignments in this Assignment Booklet were completed independently by this student.

Supervisor's Signature

If you, the student or learning facilitator, have any comments or observations regarding this module, write them in the following space.



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